Homotopy Theory
Théorie d'homotopie

(Org: **Martin Frankland** (University of Regina) and/et **Chris Kapulkin** (Western University))

TONI ANNALA, Institute for Advanced Study
Topologically protected tricolorings

Topological vortices are codimension-one topological defects that arise in various phys- ical systems, such as liquid crystals, Bose–Einstein condensates, and vacuum structures of Yang–Mills theories. Under certain homotopical assumptions that are satisfied in many realistic systems, topological vortex configurations admit faithful presentations in terms of colored link diagrams. The most well-known coloring scheme of links is given by tricolorings: each arc of the link diagram is colored by one of three possible colors (red, green, or blue) in such a way that, in each crossing, either all arcs have the same color, or all arcs have a different color. A tricolored link is topologically protected if it cannot be transformed into a disjoint union of unlinked simple loops by a sequence of color-respecting isotopies and color-respecting local cut-and-paste operations. The above operations are referred to as topologically allowed local surgeries. We use equivariant bordism groups of three-manifolds to construct invariants of colored links that are conserved in allowed local surgeries, and employ the invariant to classify all tricolored links up to local surgeries. The talk is based on joint work with Hermanni Rajamäki, Roberto Zamora Zamora, and Mikko Möttönen.

KRISTINE BAUER, University of Calgary
Faa di Bruno for bicategories

The Faà di Bruno formula is the famous formula which allows one to compute the higher derivatives of a composition of functions of a real variable. It can also be used to generalize the chain rule in differential categories (see work of Cockett and Seely, Cruttwell, and Lemay on this topic). There are a few examples of categorical differentiation which involve homotopy, most notably for abelian functor calculus (Bauer, Johnson, Osborne, Riehl and Tebbe 2018) or for infinity categories (Bauer, Burke and Ching, in progress). In this talk, I will explain how the Faa di Bruno construction in differential category theory must be expanded to be used in homotopy theory (specifically in abelian functor calculus). In particular, I will describe a program for obtaining a Faa di Bruno formula for bicategories.

PETER BUBENIK, University of Florida
Homotopy and persistent homology using closure spaces

I will develop homotopy and persistent homology in the setting of filtrations of Cech's closure spaces. Examples of filtrations of closure spaces include metric spaces, weighted directed graphs, and filtrations of topological spaces. Closure spaces have more products and intervals than topological spaces, giving us six homotopy theories, six cubical singular homology theories, and three simplicial singular homology theories. Applied to filtrations of closure spaces, these homology theories produce persistence modules. I will extend the definition of Gromov-Hausdorff distance from metric spaces to filtrations of closure spaces and use it to prove that any persistence module obtained from a homotopy-invariant functor on closure spaces is stable.

This is joint work with Nikola Milicevic.

DANIEL CARRANZA, Johns Hopkins University
Calculus of fractions for quasicategories

Calculus of fractions was introduced by P. Gabriel and M. Zisman in order to study localizations of (1-)categories. From a marked category satisfying calculus of left fractions, they construct a workable model for the localization whose morphisms are spans, rather than arbitrary zig-zags. Moreover, the localization functor preserves any finite colimits that exist in the original category.

In this talk, I will present a generalization (joint with C. Kapulkin and Z. Lindsey) of calculus of fractions to the setting of quasicategories, and show how a workable model for the localization can be constructed using a marked variant of Kan's Ex-functor. I will also discuss applications of these results to combinatorics in the form of discrete homotopy theory.

BRANDON DOHERTY, Florida State University
Cubical Joyal model structures: recent and ongoing developments

Cubical Joyal model structures, which exhibit cubical sets as a model for the theory of $(\infty, 1)$-categories, were originally developed, and shown to be equivalent to the Joyal model structure on simplicial sets, for categories of cubical sets with faces, degeneracies and connections. In this talk we will discuss the extension of these results to the category of *minimal cubical sets*, having only faces and degeneracies (arXiv:2207.03636). Time permitting, we will also discuss current work in progress on establishing cubical Joyal model structures on categories of cubical sets with diagonals and symmetries.

STERLING EBEL, University of Western Ontario
Synthetic approach to the Quillen model structure on spaces

Quillen's construction of a model structure on the category of topological spaces is a fundamental result in homotopy theory. This construction has since been applied to several related categories, such as k-spaces, and the importance of many model categories is justified by their equivalence with Quillen's structure on spaces.

In this talk, we will present an axiomatic approach to constructing Quillen's model structure on spaces to apply it to a wider range of settings. As special cases we recover several existing model structures, such as on the categories of sober spaces and of pseudotopological spaces. We also use this approach to construct a novel model structure on the category of locales, making the coreflection to sober spaces a Quillen adjunction.

This is joint work with Chris Kapulkin (arXiv:2310.14235).

ELDEN ELMANTO, University of Toronto
L-functions and algebraic K-theory

Arguably the most influential conjecture in K-theory in the previous century is Quillen-Lichtenbaum's, relating the ratio of even and odd K-groups to special values of the zeta function. This has since been resolved by Voevodsky and Rost, leading to striking development in motivic cohomology and homotopy theory. Dirichlet and Artin L-functions are generalizations of the zeta functions, where one keeps track of an additional character χ of the Galois group. One is then led to ask if these values admit similar K-theoretic interpretations. I will report on joint work with Ningchuan Zhang, where we formulate the twisted variants of the Quillen-Lichtenbaum conjectures, relating the size of K-groups with coefficients in an equivariant Moore spectrum $M(\chi)$ with special values of Artin L-functions, and prove it in some cases. I will focus on the case of finite fields where we obtain the equality:

$$Nm_{\mathbb{Q}[\chi]/\mathbb{Q}} L(-n,\chi) = \frac{\#\pi_{2n}^{C_m}\left(K(\mathbb{F}_{q^m}) \otimes M(\chi)\right)}{\#\pi_{2n-1}^{C_m}\left(K(\mathbb{F}_{q^m}) \otimes M(\chi)\right)} \qquad n \geq 1$$

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SIMON HENRY, uOttawa
Simplicial completion of model categories and strictification

Simplicial completion is a general construction to turn a combinatorial model category into a Quillen equivalent simplicial model category, by considering its category of simplicial objects. It was introduced by Cisinski in the context of Cisinski's model structure, then extended by Dugger to general left proper combinatorial model categories, and it is easy to see that if we accept to work with left semi-model categories, then it can be applied to any combinatorial model category.

But, in fact, simplicial completion can be applied to categories that are not even model categories: Given any presentable category with two cofibrantly generated weak factorization systems, we always obtain a simplicial left semi-model structure on its category of simplicial objects.

Hence an immediate question: If we start with something that doesn't have a well-defined homotopy theory, what is the homotopy theory we get after simplicial completion? i.e. what does the resulting mode category actually models? In this talk, we will answer this question and explain how this corresponds to a large generalization of Badzioch's strictification theorem to the setting of infinitary dependently typed theories.

SACHA IKONICOFF, University of Ottawa
Quillen-Barr-Beck cohomology of divided power algebras over an operad

Quillen-Barr-Beck cohomology of divided power algebras over an operad Divided power algebras are algebras equipped with additional monomial operations. They are fairly ubiquitous in the positive characteristic setting, and appear notably in the study of simplicial algebras, in crystalline cohomology, and in deformation theory. An operad is a device that encodes operations: there is an operad for associative algebras, one for commutative algebras, for Lie algebras, Poisson algebras, and so on. Each operad then comes with an associated category of algebras, and also with a category of divided power algebras.

The aim of this talk is to show how André-Quillen cohomology generalises to several categories of algebras using the notion of operad. We will introduce modules and derivations, but also representing objects for modules - known as the universal enveloping algebra - and for derivations - known as the module of Kähler differentials - which will allow us to build an analogue of the cotangent complex. We will see how these notions allow us to recover known cohomology theories on many categories of algebras, while they provide somewhat exotic new notions when applied to divided power algebras.

This is joint work with Martin Frankland and Ioannis Dokas.

ARNAB KUNDU, University of Toronto
Gersten's injectivity in the non-Noetherian world

Gersten's conjecture predicts that the K-groups of a regular local ring can be calculated by an exact sequence involving the K-groups of its residue fields. As a consequence, we may relate the K-groups of such a ring to its respective Chow groups. In this talk, we report some partial positive results to affirm the predicted injectivity part of this conjecture in a possibly mixed characteristic, non-Noetherian setting. Namely, we give evidence to show that the K-groups of an integral domain that arises as a localisation of a smooth algebra over an equi-characteristic valuation ring of rank 1 inject inside the respective K-groups of its fraction field.

UDIT MAVINKURVE, Western University
The Fundamental Group(oid) in Discrete Homotopy Theory

Discrete homotopy theory is a homotopy theory designed for studying simple graphs, detecting combinatorial, rather than topological, "holes." Central to this theory are the discrete homotopy groups, defined using maps out of grids of suitable dimensions. Of these, the discrete fundamental group in particular has found applications in various areas of mathematics, including matroid theory, hyperplane arrangements, and topological data analysis.

In this talk, based on joint work with C. Kapulkin (arxiv:2303.06029), we introduce the discrete fundamental groupoid and use it as a starting point to develop some robust computational techniques. A new notion of covering graphs allows us to extend the existing theory of universal covers to all graphs, and to prove a classification theorem for coverings. We also prove a discrete version of the Seifert-van Kampen theorem, generalizing a previous result of H. Barcelo et al. We then use it to solve the realization problem for the fundamental group through a purely combinatorial construction.

JOHN MILLER, Université de Montréal
Persistence and Triangulated Categories

The relatively new theory of Persistence Modules has seen many applications throughout geometry and topology, in particular in areas of Symplectic Geometry. In 2020 P.Biran, O.Cornea and J.Zhang introduced the notion of triangulated persistence categories (TPCs), a machinery which fits together well the theory of persistence modules with the study of geometric objects via triangulated categories.

It is common for Geometry to give rise to a Triangulated Category. However, we often forget additional data. We can recover some of this using a notion of 'persistence refinements', in which we construct a category with Hom-sets forming persistence modules and which, after localisation by a certain class of morphisms, recovers our original category. Interestingly, the combination of the persistence refinements with the triangulated structure produces a family of (pseudo)metrics on the objects of our category. These metrics and their corresponding topology seem to behave well with the underlying geometry.

The aim of this talk is to give an overview on this subject and to also discuss an algebraic problem motivated by homological mirror symmetry; how to extend these metrics to the Karoubi completion.

RACHEL HARDEMAN MORRILL, University of Calgary
Path Categories and Graphs

In classical homotopy theory, two spaces are considered homotopy equivalent if one space can be continuously deformed into the other. This theory, however, does not respect the discrete structure of graphs with their vertices and edges. For this reason, a discrete homotopy theory for graphs is needed. A path category is a structure associated to Moore paths and a natural starting place for defining a homotopy theory. In this talk, I will discuss what a path category is, how path categories can be used to define a discrete homotopy theory for graphs, and what kind of structure a path category gives. This work was done in collaboration with Laura Scull and Robin Cockett.

DORETTE PRONK, Dalhousie University
Double Category Sites for Grothendieck Topoi

In (Cahiers, 2020) DeWolf and I introduce a type of double categories with an Ehresmann topology as sites for étendues. This extends work by Lawson and Steinberg for ordered groupoids: we reinterpret them as double categories and establish a 2-adjunction between the 2-categories of Ehresmann sites and left-cancellative Grothendieck sites. However, since the category of left-cancellative sites does not satisfy the Ore condition for the class of Comparison Lemma maps, we cannot represent geometric morphisms between étendues as a bicategory of fractions for these sites.

In this talk I will introduce a generalization for the notion of Ehresmann site that can be used to represent any Grothendieck topos. The corresponding class of Grothendieck sites is that of the covering-mono sites: sites for which the single arrow coverages and the monics form an orthogonal factorization system. We show that the Comparison Lemma maps do give a calculus of fractions for this class of sites. The corresponding Ehresmann site double category has covering arrows as horizontal arrows and inclusions of subobjects as vertical arrows. We introduce a notion of covering-flat and covering preserving double functors between these Ehresmann sites with a class of Comparison Lemma double functors, that lets us express geometric morphisms in terms of fractions. This presentation restricts to étendues by only considering the so-called torsion-free generated sites. If time permits I will discuss how this can be used to represent orbifolds. This is joint work with Darien DeWolf (St Francis Xavier University) and Julia Ramos González (UC Louvain).

NICK ROZENBLYUM, University of Toronto
Stratifications and reflection

I will describe a new duality in homotopy theory called reflection. This duality simultaneously generalizes Greenlees-May duality, the theory of BGP reflection functors giving derived equivalences of quivers, and Lurie's ∞-categorical Dold-Kan correspondence. Moreover, it leads to a categorification of the classical Mobius inversion formula. It is based on a theory of stratified (stable presentable) ∞-categories and is closely related to Verdier duality on stratified spaces. This is joint work with David Ayala and Aaron Mazel-Gee.

TSELEUNG LARRY SO, University of Western Ontario
The cohomology of 4-dimensional toric orbifolds

A toric orbifold is an even dimensional orbifold formed by gluing tori together in a combinatorial way determined by a simple polytope and a characteristic function. Examples include weighted projective spaces and quasitoric manifolds.

In 2006 Masuda and Suh posed the cohomological rigidity problem: in which class of toric spaces the geometric / topological structures of the spaces are determined solely by their cohomology rings? It has become one of the major and longstanding problems in toric topology. Extensive work has been done to prove an affirmative answer in several smooth cases and to investigate various adaptions of the problem. In the orbifold case, the cohomology rigidity problem is even more challenging and very little is known.

In addition, despite being fundamental objects in toric topology, the cohomology rings of toric orbifolds remain largely unknown except for very few special cases. A central problem is to understand the interplay between their cohomology ring structures and the underlying combinatorial data.

In my presentation I will talk about my joint projects on the cohomological rigidity and the cohomology ring structures of 4-dimensional toric orbifolds.

This is joint work with Xin Fu (Beijing Institute of Mathematical Sciences and Applications) and Jongbaek Song (Pusan National University).

DON STANLEY,
Which graded algebras are the cohomology of a space?

This is an old problem sometimes called Steenrod's problem. Important examples include Hopf invariant 1 resolved by Adams and a complete solutions over the rationals by Quillen. We describe some recent progress on the problem for algebras over the integers that become (graded) exterior after tensoring with the rationals. This is joint work with Larry So and Stephen Theriault.

CARLOS GABRIEL VALENZUELA, University of Regina
Double cohomology and sphere triangulations

Given a simplicial complex $\mathcal{K}$ it's of general interest to study it's moment-angle complex $\mathcal{Z}_{\mathcal{K}}$, particularly in toric topology. In 2020, Limonchenko, Panov, Stanley and Song designed a new homological invariant for $\mathcal{Z}_{\mathcal{K}}$ called the double (co)homology. This invariant is less chaotic by design than the regular (co)homology of $\mathcal{Z}_{\mathcal{K}}$ which can be of interest for applications. It remains an unsolved problem whether this new (co)homology theory can be of any prescribed rank.

In my work I've been studying sphere triangulations and how their associated double cohomology behaves under operations to them. These are particularly interesting, as their double cohomology is a Poincare Algebra. Furthermore, these complexes turn out to be a good starting point for constructing complexes with exotic double homology rank.

During the talk I'll introduce the construction of double homology and present some recent results about it.

JERRY WEI, University of Toronto
Analogies of Lie Group Concepts in S^7 and the Space of Commuting Pairs

The octonions, also known as Cayley numbers, has a norm-preserving multiplication. The unit octonions S^7 is an H-space which is not a Lie group due to failure of associativity. We examine the extent to which S^7 does or does not have analogies to Lie group concepts such as maximal torus, Weyl group, etc. Moreover, we give an explicit description of the centralizer of elements of S^7 and use it to compute the homology of the space of commuting pairs $\{(x, y) \in S^7 \times S^7 : xy = yx\}$.

BEN WILLIAMS, UBC
Looking for extraordinary involutions

If X is a topological space with a self-map $\lambda : X \to X$ of order 2 and A is a bundle of matrix algebras (a topological Azumaya algebra) over X, then a λ-involution of A is an isomorphism of bundles of algebras $\sigma : A \to \lambda^* A^{\mathrm{op}}$. The structure (A, σ) may be locally isomorphic to a trivial algebra $\mathrm{Mat}_{n\times n}(\mathbb{C}) \times X$ with some involution, in which case σ is said to be of ordinary type. In this talk, I will explain the algebraic topology underlying the construction of more exotic examples of involutions.