Student Research Session
Séance de recherche étudiante
(Org: **Karen Julia Fletcher** (Athabasca University) and/et **Daniel Zackon** (McGill University))

ADRIANA-STEFANIA CIUPEANU,
Dynamics of Variants of Concern

Abstract: The COVID-19 pandemic has seen multiple waves, in part due to the implementation and relaxation of social distancing measures by the public health authorities around the world, and also caused by the emergence of new variants of concern (VOCs) of the SARS-Cov-2 virus. Using mathematical modelling tools, we investigated the dynamics of VOCs with the objective of understanding key factors that determine the dominance and coexistence of VOCs. Our results show that the transmissibility advantage of a new VOC is a main factor for it to become dominant. Additionally, our modelling study indicates that the initial number of people infected with the new VOC plays an important role in determining the size of the epidemic. Furthermore, public health measures targeting the newly emerging VOC taken in the early phase of its spread can limit the size of the epidemic caused by the new VOC. This is joint work with Marie Varughese, Weston Roda, Donglin Han, Qun Cheng, Michael Y. Li

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SHREYA DHAR, CHENGLU WANG, GRAYSON PLUMPTON & RIVER NEWMAN, U of T (Shreya), Penn (Chenglu), Queens (Grayson), Yale (River)
On the Classification of Field Extensions of p-adic Fields

Let p be a prime and let $\mathbb{Q}_p$ be the field of p-adic numbers. It is known that the number of finite extensions of $\mathbb{Q}_p$ of a given degree is finite up to isomorphism. Given a finite algebraic extension L of $\mathbb{Q}_p$ generated by the root of an irreducible polynomial h, we present a practical (closed-form) method to determine the isomorphism class in which L lives, based on the coefficients of h. We will discuss the subtleties of the case when the degree of the extension coincides with p, the characteristic of the residue field.

MERAJ HOSSEINI, Concordia University
Contraction of Convex Hypersurfaces in $\mathbb{R}^3$ by Powers of Principal Curvatures

We study the contraction of strictly convex, axially symmetric hypersurfaces by a non-symmetric, non-homogeneous, fully nonlinear function of curvature. Starting from axially symmetric hypersurfaces with even profile curves, we show evolving hypersurfaces converge to a single point in a finite time, and under proper rescaling, solutions will converge to a convex hypersurface.

ALEXANDER KROITOR, University of Waterloo
Asymptotics For Lattice Paths Through Analytic Combinatorics

The topic of lattice path enumeration finds applications in a variety of areas, from queuing theory to statistical physics. Recent work on the behaviour of lattice walks restricted to quadrants combines a combinatorial tool called the kernel method with techniques from the field of analytic combinatorics in several variables (ACSV) to determine asymptotics of a large variety of models. In this talk we describe new results using this framework, going beyond past work by analyzing more pathological saddle-point integrals using new complex analytic techniques.

CHRISTOPHER JAMES LANG, University of Waterloo
Spherically symmetric hyperbolic monopoles

Hyperbolic monopoles are solutions to the Bogomolny equations, a system of partial differential equations on hyperbolic 3-space, though few examples exist. Using a correspondence with circle-invariant instantons, we reduce the problem of finding spherically symmetric hyperbolic monopoles (solving differential equations) to a problem in the realm of representation theory, providing much needed examples of these objects.

HAGGAI LIU, Simon Fraser University
Moduli Spaces of Weighted Stable Curves and their Fundamental Groups

The Deligne-Mumford compactification, $\overline{M_{0,n}}$, of the moduli space of n distinct ordered points on $\mathbb{P}^1$, has many well understood geometric and topological properties. For example, it is a smooth projective variety over its base field. Many interesting properties are known for the manifold $\overline{M_{0,n}}(\mathbb{R})$ of real points of this variety. In particular, its fundamental group, $\pi_1(\overline{M_{0,n}}(\mathbb{R}))$, is related, via a short exact sequence, to another group known as the cactus group. Henriques and Kamnitzer gave an elegant combinatorial presentation of this cactus group.

We study a weighted variant of $\overline{M_{0,n}}(\mathbb{R})$ known as a Hassett space: For each of the n labels, we assign a weight between 0 and 1; points can coincide if the sum of their weights does not exceed one. Our goal is to find combinatorial presentations for the fundamental groups of Hassett spaces with certain restrictions on the weights. To proceed with our goal, we use two main approaches: The first approach is to recursively compute them using blowups, Seifert Van-Kampen, and knowledge for smaller n. The second approach is to express the Hassett space as a blow-down of $\overline{M_{0,n}}$ and modify the cactus group directly.

FADIA OUNISSI, Concordia University
On Rogers-Shephard type inequalities for (n-1)-dimensional volumes

The difference body $K - K$ of a convex body $K \in \mathbb{R}^n$, formed by taking the Minkowski sum of K and $-K$, has been extensively studied, notably following the conjectured Rogers-Shephard inequality, $V_n(K-K) \leq \binom{2n}{n} V_n(K)$, with equality if and only if K is a simplex. Although this inequality holds for n-dimensional volumes, very little is known about the upper bound for the $(n-1)$-dimensional volume of $K-K$. In this talk, we first introduce some preliminaries on convex analysis, and discuss our asymptotic upper bound for $\frac{V_{n-1}(K-K)}{V_{n-1}(K)}$, supported by estimates for some classes of polytopes and nonsmooth bodies in $\mathbb{R}^3$.

CHAABANE REJEB, Université de Sherbrooke
Quasi-homogeneous solutions to the WDVV equations associated with the genus one Hurwitz-Frobenius manifolds.

We consider the genus one Hurwitz space of ramified coverings of the Riemann sphere with prescribed ramification profile over the point at infinity. We construct on Hurwitz spaces a family of Frobenius manifold structures associated with the quasi-homogeneous differentials. We explicitly derive new generating formulas for the corresponding prepotentials. This produces quasi-homogeneous solutions to the following generalized WDVV associativity equations: $F_i \eta^{-1} F_j = F_j \eta^{-1} F_i$, where the invertible constant matrix η is a linear combination of the matrices F_j. As applications, we obtain explicit solutions to the WDVV equations in genus one and give a new proof of Ramanujan's differential equations for the Eisenstein series E_2, E_4 and E_6.

NAHID SADR, Université de Sherbrooke
Index-mixed copulas

Copulas provide a framework for modeling dependence between random variables. They are particularly important in multivariate statistics and risk management, as they help model the relationship between variables while accounting for their marginal distributions. In this talk, we aim to introduce the basics of copula theory, how copulas relate to multivariate joint distributions via the celebrated Sklar's theorem, and compare some families of copulas studied in the literature that are used in theory and practice to capture different dependence scenarios. Afterwards, our research on a new class of copulas named index-mixed copulas is introduced, and its properties are investigated. Index-mixed copulas are constructed from given base copulas and a random index vector, and show a rather remarkable degree of analytical tractability. The analytical form of the copula and, if it exists, its density is derived. As the construction is based on a stochastic representation, sampling algorithms can be given. Properties investigated include bivariate and trivariate margins, tail dependence, measures of concordance such as Spearman's rho or Kendall's tau, and concordance orderings. A particularly interesting feature of index-mixed copulas is that they allow one to provide an interpretation of the well-known family of Eyraud-Farlie-Gumbel-Morgenstern (EFGM) copulas, which are popular for their analytical tractability. Through the lens of index-mixing, one can see EFGM copulas can only model a limited range of concordance and are tail independent, but this is not the case for index-mixed copulas in general.

SCOTT WESLEY, Dalhousie Unviersity
Towards an Algebraic and Geometric Theory of Quantum Circuits

The circuit diagrams studied in computer science enjoy a rich mathematical theory. Given a finite set of primitive operators (known as "gates"), a circuit diagram is any operator obtained by composing finitely many gates in sequence or in parallel. Formally, circuits correspond to string diagrams in finitely-generated monoidal categories. A special class of circuit diagrams are the classical reversible circuits, in which gates are invertible matrices over $\mathbb{Z}_2$. It was shown by Toffoli in 1980 that every classical reversible circuit is constructible from a single primitive known as the Toffoli gate. More generally, one can study monoidal groupoids, which characterize all reversible models of computation. For example, the reversible quantum circuits studied by Feynman correspond to the monoidal groupoid of unitary matrices. Since unitary matrices are uncountable, there does not exist an exact universal gate set for quantum computation. However, given both the Toffoli and Hadamard gate, all unitary operators can be simulated.

This talk begins with an introduction to combinatorial circuits as symmetric monoidal string diagrams. The case of classical reversible circuits is recalled. It is then shown how quantum mechanics gives rise to a groupoid of reversible circuits subsuming the classical case. The homsets in this category form groups of circuits with identical wire counts. Presentations for these groups can answer many questions in quantum computing. As a specific example, the 3-qubit dyadic Toffoli+H circuits are considered, whose presentation emerges from the E8 lattice. This presentation, in turn, yields information about the entire sub-groupoid of dyadic Toffoli+H circuits.

TONATIUH MATOS WIEDERHOLD, University of Toronto
The lattice of uniform topologies

Given a Tychonoff space X (e.g., the reals), there are many ways to topologize $C(X)$, the space of continuous functions from X to $\mathbb{R}$. If we order a specific family of uniform topologies appropriately, we get an atomic complete lattice with many interesting properties, some of which can be used to study the original space X itself. In this talk, we give an introduction to this lattice, discuss some curious properties, open problems and recent discoveries, followed by an invitation to explore this exciting new field.

This is joint work in progress with Dr. Roberto Pichardo Mendoza.