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*Risk-Sensitive Control and Mean Field Games: A Variational Approach*

We develop a variational approach to address risk-sensitive optimal control problems with an exponential-of-integral cost functional in a general linear-quadratic-Gaussian (LQG) single-agent setup, offering new insights into such problems. Our analysis leads to the derivation of a nonlinear necessary and sufficient condition of optimality, expressed in terms of martingale processes. Subject to specific conditions, we find an equivalent risk-neutral measure, under which a linear state feedback form can be obtained for the optimal control. It is then shown that the obtained feedback control is consistent with the imposed condition and remains optimal under the original measure. Building upon this development, we (i) propose a variational framework for general LQG risk-sensitive mean-field games (MFGs) and (ii) advance the LQG risk-sensitive MFG theory by incorporating a major agent in the framework. The major agent interacts with a large number of minor agents, and unlike the minor agents, its influence on the system remains significant even with an increasing number of minor agents. We derive the Markovian closed-loop best-response strategies of agents in the limiting case where the number of agents goes to infinity. We establish that the set of obtained best-response strategies yields a Nash equilibrium in the limiting case and an  $\varepsilon$ -Nash equilibrium in the finite-player case.