SEDANUR ALBAYRAK, University of Calgary
Quantitative estimates for the size of an intersection of sparse automatic sets

In this talk, I will talk about use of automata theory in answering problems in number theory. In 1844, Catalan conjectured that the set consisting of natural numbers of the form $2^n + 1$, $n \geq 0$ and the set consisting of powers of 3 has finite intersection. In fact, we can answer such question in more generality, that is, instead of 2 and 3, we can show this for k and ℓ that are multiplicatively independent (meaning if $k^a = \ell^b$, then $a = b = 0$). In automata-theoretic terms, these sets described above are sparse 2-automatic and sparse 3-automatic sets, respectively. In fact, a sparse k-automatic set can be more complicated than having elements that are of the form k^n or $k^n + 1$, and hence, we are answering a more general question. Moreover, we also prove our result in a multidimensional setting in line with the existing results in the theory of formal languages and finite automata. We show that the intersection of a sparse k-automatic subset of $\mathbb{N}^d$ and a sparse ℓ-automatic subset of $\mathbb{N}^d$ is finite and we give effectively computable upper bounds on the size of the intersection in terms of data from the automata that accept these sets.