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SYSTEMATIC SEARCH FOR EXTREME BEHAVIOUR IN 3D NAVIER-STOKES EQUATIONS BASED ON THE LADYZHENSKAYA-PRODI-SERRIN CONDITIONS

This investigation concerns a systematic search for potential singularities in 3D Navier-Stokes flows. It is based on the Ladyzhenskaya-Prodi-Serrin conditions, which assert that if the quantity $\int_0^T \|\mathbf{u}(t)\|_{L^q(\Omega)}^p \, dt$ remains bounded, given that $2/p + 3/q \leq 1$ and $q > 3$, then the solution $\mathbf{u}(t)$ of the Navier-Stokes system remains smooth within the interval $[0, T]$. Hence, should a singularity arise at any instant within the interval $[0, T]$, we would anticipate an unbounded growth of this quantity.

We examine these conditions by solving numerically a set of variational optimization problems. These problems aim to determine initial conditions $\mathbf{u}_0$ such tat the corresponding flow maximizes $\int_0^T \|\mathbf{u}(t)\|_{L^q(\Omega)}^p \, dt$ for different values of T while satisfying specific constraints. We address these problems computationally, employing a large-scale adjoint-based gradient approach in Sobolev and Lebesgue spaces.

We extend earlier work by considering various values of q, and different types of gradients to discretize gradient flows. We also studied the limiting case $q = 3$ where the regularity condition is slightly different.