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Complex Analytic Structure of Stationary Solutions of the Euler Equations

This work is devoted to the stationary solutions of the 2D Euler equations describing the time-independent flows of an ideal incompressible fluid. There exists an infinite-dimensional set of such solutions. Prevous authors considered the solutions in the Frechet space of smooth functions and used powerful methods such as the Nash-Moser-Hamilton implicit function theorem to provide a smooth local parameterization of stationary flows in annular domains without stagnation point. However, in their approach they overlook a surprising feature of the stationary flows which makes the picture much more transparent, and opens the way to further progress. This is the observation that the flow lines are analytic curves, despite limited regularity of the velocity field.

To study the stationary flows we change the viewpoint and consider the flow field as a family of analytic flow lines non-analytically depending on parameter. We quantify the analyticity by introducing spaces of functions which have analytic continuation in one of the variables to a complex strip containing the real axis, with appropriate behaviour on the strip boundary. These partially-analytic functions form a complex Banach space. The stationary solutions satisfy (in the new coordinates) a quasilinear elliptic equation, degenerating in the presence of a stagnation point. Local solvability is proved by using the Banach analytic implicit function theorem. Thus we prove that the set of stationary flows is locally a complex analytic Banach manifold.