Geometry in Calculus of Variations and PDEs

(Org: **Lia Bronsard** and/et **Dominik Stantejsky** (McMaster University))

TIZIANA GIORGI, The University of Alabama

SmA-type phases of bent-core liquid crystals

We will present some analytical and computational results on the modeling and simulation of polar smectic A phases formed by rigid bent-core molecules. This is joint work with Carlos García-Cervera and Sookyung Joo.

DIANE GUIGNARD, University of Ottawa

Finite Element Methods for the Stretching and Bending of Thin Structures with Folding

We study the elastic behavior of prestrained thin plates which can undergo large deformations and achieve non-trivial equilibrium shapes even without external forces. The mathematical problem consists in minimizing an energy of the form $E(\mathbf{y}) = E_S(\mathbf{y}) + \theta^2 E_B(\mathbf{y})$, where $\mathbf{y}$ is the deformation of the midplane, θ is the thickness of the plate, E_S is the (nonconvex) stretching energy, and E_B is the bending energy.

We introduce a discrete energy based on a continuous finite element space and a discrete Hessian operator involving the jump of the gradient of the deformation across the interelement sides. We establish the Γ-convergence of the discrete energy and also present an energy-decreasing gradient flow for finding critical points of the discrete energy. We provide numerical simulations illustrating the capabilities of the model.

This is joint work with A. Bonito and A. Morvant (Texas A&M).

KENNEDY IDU, University of Toronto

On the Alexandrov's estimate

A classical fact due to Alexandrov states that if Ω is a bounded open convex domain in $\mathbb{R}^n$, and $u : \bar{\Omega} \to \mathbb{R}$ is a convex function such that $u = 0$ on $\partial\Omega$, then

$$[u]^n_{1/n} \leq C(\Omega)|\partial u(\Omega)|.$$

Here ∂u denotes the subgradient of u. The estimate is not only crucial to regularity theory of the Monge-Ampere equation, but also main tool in some linear elliptic PDE estimates. In this talk, will discuss some extensions and refinements of the estimate using the geometry of $\partial\Omega$. This is a joint work with Charles Griffin and Robert L. Jerrard (University of Toronto).

XIN YANG LU, Lakehead University

A physicality-enforcing convex singular potential

Liquid crystals (LC) are an intermediate state of the matter between solids and liquids, exhibiting significant mobility, but also having a preferred orientation, commonly referred to as "director". LCs themselves exhibit several phases, e.g. nematic, smectic, chiral/twisted, discotic, conic. Nematic LCs are the simplest ones, being characterized by only a director. Modeling LCs has been a long standing problem. One of the most widely models is the Landau-de Gennes theory. In 3D, the main quantity is a 3×3 Q-tensor matrix. Due to modeling requirements, the eigenvalues of the Q-tensor must be constrained in $(-1/3, 2/3)$, a condition known as "physicality". One way to enforce this is to add a singular convex potential ψ, introduced by Ball and Majumdar. Powerful from a theoretical point of view, such ψ is defined only implicitly, as the integral of an entropy-like term, making its analysis quite challenging. In this talk, we will present several crucial estimates on ψ and its derivatives.

LORENA AGUIRRE SALAZAR, Lakehead University

On a relationship between the TFDW and the Liquid Drop models via Gamma convergence

We consider the TFDW model and the Liquid Drop Model with external potential, a model proposed by Gamow in the context of nuclear structure. It has been observed that the TFDW model and the Liquid Drop Model exhibit many of the same properties, especially in regard to the existence and nonexistence of minimizers. We show that, under a "sharp interface" scaling of the coefficients, the TFDW energy with constrained mass Gamma-converges to the Liquid Drop model, for a general class of external potentials. Finally, we present some consequences for global minimizers of each model

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DOMINIK STANTEJSKY, McMaster University
On Minimizing Harmonic Maps with Planar Boundary Anchoring

Motivated by experiments with nematic liquid crystal droplets, we study harmonic maps that arise as minimizers of the the one-constant approximation of the Osee-Frank energy subject to strong anchoring tangential boundary condition. In this talk, I will present a reflection method allowing us to analyze the regularity of minimizers up to the boundary. We also obtain results on the type and location of defects that can occur, such as boundary "boojums" and interior vortices. The talk is based on joint work with L. Bronsard and A. Colinet.

JACK TISDELL, McGill University
Minimizing asymptotic score in random bullseye darts for i.i.d. throws

We present current work—motivated by considerations of the energy of random Voronoi diagrams—on the score in a certain game of darts with both a random bullseye and random throws. We discuss the convergence properties as the number of throws tends to infinity and the asymptotically optimal distribution for the player assuming i.i.d. throws. Curiously, the moments of the score under under optimal play in this sense bear a simple relation to the asymptotically optimal quantizers of the bullseye distribution.

IHSAN TOPALOGLU, VCU
Minimizing sets of weakly-repulsive nonlocal energies

In this talk we will consider weakly repulsive-strongly attractive nonlocal interaction energies over bounded densities of fixed mass m. In particular, we will show that under certain regularity assumptions on the interaction kernels these energies admit minimizers given by characteristic functions of sets volume m when m is sufficiently small (or even for every m, in some cases). Finally, we will present on a generalization of a recent result of Davies, Lim and McCann, and give sufficient conditions that guarantee that minimizers over probability measures are given by Dirac masses concentrated on the vertices of a regular $(N+1)$-gon in $\mathbb{R}^N$. This is a joint work with Davide Carazzato and Aldo Pratelli.