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Uniform hypergraphs and balanced incomplete block designs with r-friendship property

A t-uniform hypergraph $\mathcal{H}$ has r-friendship property if for every t-subset of vertices $v_1, \ldots, v_t$, there are exactly r vertices $w_1, \ldots, w_r$ such that for $(t-1)$-subsets A of vertices v_i, and any w_i, $A \cup \{w_i\}$ is a hyperedge in $\mathcal{H}$. Li et al. conjectured that no balanced incomplete block design (BIBD) has 1-friendship property. We show that if $\mathcal{H}$ is a 1-friendship t-uniform hypergraph that is a BIBD-(n, b, d, t, λ), then n is small enough with respect to λ. Furthermore, we present a class of 1-friendship t-uniform hypergraph that is a BIBD. We generalize our results to r-friendship t-uniform hypergraph and show no such hypergraphs are BIBD when n is large enough with respect to λ, t, and r.