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Nodal topology and persistence barcodes

Classical Courant's nodal domain theorem, together with Weyl's law, gives an upper bound on the number of nodal domains of a Laplace-Beltrami eigenfunction in terms of the corresponding eigenvalue. In general, bounds of this type can not exist for linear combinations of eigenfunctions. We will show how, by coarsely counting nodal domains, i.e. by discarding small oscillations, we may obtain a similar upper bound for linear combinations as well. Our method combines the theory of persistence modules and barcodes with multiscale polynomial approximation of functions in Sobolev spaces. Using the same method, we may study coarse topology of a zero set of a function, as well as coarse topology of the set of common zeros of a number of different functions. This allows us to prove a coarse version of Bézout's theorem for linear combination of Laplace-Beltrami eigenfunctions. The talk is based on a joint work with L. Buhovsky, J. Payette, I. Polterovich, L. Polterovich and E. Shelukhin.