Commutative Algebra
Algèbre commutative

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NASRIN ALTAFI, Queen's University

The Weak Lefschetz property and the number of generators of equigenerated monomial ideals

For a polynomial ring R with standard grading over a field k, we say a graded Artinian quotient algebra R/I has the weak Lefschetz property (WLP) if multiplication by a general linear form has maximal rank in every degree. Over a field k of characteristic zero, I will discuss sharp bounds on the number of generators of an equigenerated monomial ideal I for which R/I satisfies the WLP. This is joint work with Samuel Lundqvist.

KIERAN BHASKARA, McMaster University

Regularity and projective dimension of toric ideals of bipartite graphs

The regularity and projective dimension of combinatorially-defined ideals are frequently-studied invariants in combinatorial commutative algebra. In particular, much work has been done towards understanding the values these invariants can achieve for toric ideals I_G associated with a graph G. In this talk, we fully describe the possible values of these invariants for I_G as G ranges over all bipartite graphs on a fixed number of vertices. As a corollary, we show that any pair of positive integers can be realized as the regularity and projective dimension of a toric ideal of a bipartite graph. Finally, we demonstrate how our main result allows us to completely determine the values all five major invariants studied in the literature for this family of graphs.

TRUNG CHAU, University of Utah

Barile-Macchia resolutions for monomial ideals

We develop an algorithm to create homogeneous acyclic matchings for any given monomial ideal. Via discrete Morse theory, they induce cellular resolutions for this ideal, which we call Barile-Macchia resolutions. These resolutions are minimal for edge ideals of weighted oriented forests and (most) cycles. As a result, obtain recursive formulas for graded Betti numbers and projective dimension. Furthermore, we compare Barile-Macchia resolutions to those created by Batzies and Welker and some well-known simplicial resolutions. Under certain assumptions, whenever the above resolutions are minimal, so are Barile-Macchia resolutions.

SUSAN COOPER, University of Manitoba

Resolutions & Powers of Ideals

Diana Taylor established a technique to construct a free resolution of an ideal I generated by s monomials using the simplicial chain maps of a simplex on s vertices. Work of Bayer, Peeva and Sturmfels later extended Taylor's work to show that as long as such a simplicial complex satisfies certain homological conditions, it can support a free resolution of I. The idea of using the structure of I to further find data about the Betti numbers of powers of I becomes quite delicate quickly. In this talk we consider a square-free monomial ideal I and describe a complex labeled with the generators of I^r which supports a free resolution of I^r.

MIKE CUMMINGS, McMaster University

A Gröbner basis for regular nilpotent Hessenberg Schubert cells

Hessenberg varieties lie at the intersection of algebraic geometry, combinatorics, and topology. A trend in the last decade has been to study Hessenberg varieties via their local defining ideals, called patch ideals. Recently, Da Silva and Harada showed that in the regular nilpotent case, the patch ideal of the longest word permutation w_0 is a particularly nice complete intersection, which they called a triangular complete intersection.

In this talk, we will define triangular complete intersections and discuss several nice applications, including to Hilbert series and Gröbner bases. We will also show how Da Silva and Harada's w_0-patch results translate to arbitrary patches for the local defining ideals of intersections of regular nilpotent Hessenberg varieties with Schubert cells. This translation preserves triangular complete intersections and we recover Tymoczko's result—in type A—that regular nilpotent Hessenberg varieties are paved by affines.

This is based on work with Sergio Da Silva, Megumi Harada, and Jenna Rajchgot.

THIAGO DE HOLLEBEN, Dalhousie University

Rees algebras and Lefschetz properties of squarefree monomial ideals

Many interesting algebraic properties of edge ideals of graphs can be checked by simply verifying whether the underlying graph is bipartite or not. Some examples of this include: the linear type property, the birational property of the rational map the ideal defines, and the equality of symbolic and ordinary powers. From a graph theoretic perspective, a graph (with more edges than vertices) is not bipartite if and only if its incidence matrix has full rank. In this talk, we will see how these equivalences help in the study of Lefschetz properties. We will also see how the study of Lefschetz properties may bring new perspectives into the study of these algebraic properties. In particular, we will see hints of connections between symbolic defects of squarefree monomial ideals and f-vectors of simplicial complexes.

ELENA GUARDO, Università di Catania

Expecting the unexpected: quantifying the persistence of unexpected hypersurfaces

Let X be a reduced subscheme in $\mathbb{P}^n$. We say that X admits an unexpected hypersurface of degree d and multiplicity m if the imposition of having multiplicity m at a general point P fails to impose the expected number of conditions on the linear system of hypersurfaces of degree d containing X. We introduce new methods for studying unexpectedness, such as the use of generic initial ideals and partial elimination ideals to clarify when it can and when it cannot occur. We formulate a new way of quantifying unexpectedness (our AV sequence), which allows us detect the extent to which unexpectedness persists as increases but remains constant. We also study how knowledge of the Hilbert function, together with certain geometric assumptions, can provide information about unexpected hypersurfaces.

ARVIND KUMAR, New Mexico State University

Resurgence of Classical Varieties

Resurgence number measures the containment behavior of symbolic powers of an ideal into its ordinary powers. However, in general, it does not always resolve the containment problem. In this talk, we will see that resurgence completely resolves the containment problem for classical varieties. This talk will be based on an ongoing work with Vivek Mukundan.

PEILIN LI, University of British Columbia

Building Monomial Ideal with Fixed Betti Number

A minimal free resolution of an ideal produces a sequence of integers which are called "Betti numbers." In this talk, I will introduce a constructive method to add generators to a monomial ideal I while preserving most Betti numbers of I. The main method we use to find such monomials is simplicial collapsing from algebraic topology. I will start with an introduction on simplicial collapsing - making a simplicial complex smaller by deleting some faces – and how it interacts with free resolutions of monomial ideals. In my talk, I will show how one can change a monomial ideal, one generator at a time, and keep track of the Betti numbers at the same time using simplicial collapses. Furthermore, with this method, starting with a monomial ideal I, and by a sequence of operations, I will show that we can create infinitely many monomial ideals with arbitrarily many generators that have similar Betti numbers as I. And we can also create infinitely many monomial ideals with the same number generators as I which have exactly the same Betti numbers as I.

IRESHA MADDUWE, Dalhousie University
Reconstruction Conjecture on Homological Invariants of Cameron Walker Graphs

We will discuss that the homological invariants of edge ideals of Cameron Walker graphs, such as regularity, and depth can be reconstructed from its vertex deleted subgraphs. Moreover, we will speak on reconstruction of the lattice points of the edge ideals of Cameron Walker graphs such as $(\mathrm{reg}(R/I), \deg h(R/I))$ and $(\mathrm{depth}(R/I), \dim(R/I))$ using the lattice points of their vertex-deleted subgraphs.

HASAN MAHMOOD, Dalhousie University
Mutation of Simplicial Complexes

When a brain processes some information at a certain time, this activity of the brain can be described as a simplicial complex whose vertices are the neurons. This simplicial complex will undergo a change for performing another activity depending on what information is being processed - and the process of changing one simplicial complex to another continues. Let us generalize this phenomenon. Let Δ be a simplicial complex on a finite vertex set V. In theory, there can be many ways of obtaining a simplicial complex from the given one. Let us fix one such way and call it $\mathcal{T}$, which transforms Δ into a new simplicial complex $\mathcal{T}(\Delta)$. Set $\Delta_k = \mathcal{T}(\Delta_{k-1})$ with setting $\Delta_0 = \Delta$. This way we obtain a sequence $\{\Delta_k : k \in \mathbb{N}\}$ of simplicial complexes. Let us call this sequence a mutation of Δ under $\mathcal{T}$. Since there can only be a finite number of simplicial complexes on V, this mutation would either stabilize, "vanish", or start repeating its terms (up to isomorphism) at some point. In my talk, I will talk about the stability of mutations of some simplicial complexes for a special $\mathcal{T}$ that returns us the Stanley-Reisner complex of the facet ideal for the input simplicial complex.

MICHAEL MORROW, University of Kentucky
Syzygy Computations in OI-Modules

Given a sequence of related modules M_n defined over a sequence of related polynomial rings, one may ask how to simultaneously compute the syzygy module of each M_n. Working in the setting of OI-modules over Noetherian polynomial OI-algebras, we present an OI-analogue of Schreyer's theorem for computing syzygies. Here, OI denotes the category of totally ordered finite sets with order-preserving injective maps.

RITIKA NAIR, University of Kansas
An Improved Terai-Yoshida Theorem

In 2005, Terai-Yoshida showed that certain class of Stanley-Reisner rings having sufficiently large multiplicities are Cohen-Macaulay. In the joint work with Anton Dochtermann, Jay Schweig, Adam Van Tuyl and Russ Woodroofe, we strengthen their result by showing that simplicial complexes having many facets are vertex decomposable. If time permits, we shall also discuss an alternative proof for the Alexander dual version of the Terai-Yoshida result.

SHAH ROSHAN-ZAMIR, University of Nebraska-Lincoln
Interpolation in the Weighted Projective Space

Given a finite set of points X in the projective space over a field k one can ask for the k-vector space dimension of all degree d polynomials that vanish to order two on X. (These are polynomials whose first derivative vanishes on X.) The Alexander-Hirschowitz theorem (A-H) computes this dimension in terms of the multiplicity of the points and the k-vector space dimension of degree d monomials, with finitely many exceptions. In this talk, we investigate this question in the weighted projective line and space, $\mathbb{P}(s,t)$ and $\mathbb{P}(a,b,c)$. We define a notion of multiplicity for weighted spaces, give an example of $\mathbb{P}(a,b,c)$ where A-H holds with no exceptions and an infinite family where A-H fails for even one point, and discuss future directions.

SERGIO DA SILVA, Virginia State University

Cohen-Macaulay Toric Ideals of Graphs and Geometric Vertex Decomposition

Understanding when the toric ideal of a graph defines a Cohen-Macaulay variety remains an open problem which is related to whether the ideal is geometrically vertex decomposable (GVD). Toric ideals of graphs which are GVD are automatically glicci and Cohen-Macaulay, but it can be difficult to check the GVD property directly. Alternate versions of the property, like being weakly GVD or GVD allowing substitution, are easier to check and still imply the Cohen-Macaulay property. I will provide a brief overview of geometric vertex decomposition and how its various formulations can be used to find graphs whose toric ideals are Cohen-Macaulay. I will also provide an update on topics related to the interaction of GVDs with toric ideals of graphs, including graph coloring, Hamiltonian cycles, and the GVD classification problem.