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Pseudoku: A Sudoku Adjacency Algebra and Fractional Completion Threshold

We develop a new 4-partite graph representation, G_P , for a partial Sudoku, P . In this representation, partite sets correspond to the rows, columns, boxes, and symbols of P . The edges represent unfulfilled conditions in P that are necessary for a completed Sudoku. For instance, if symbol 3 is missing from the first row in P , an edge is drawn between those vertices in G_P . We define a tile to be a 4-vertex subgraph of G_P corresponding to a valid placement of a symbol in P . We note that P can be completed if and only if its graph representation G_P has an edge-decomposition into tiles. We then relate the existence of a tile-decomposition to the existence of a solution to a specific linear system using an edge-tile inclusion matrix. Through an in-depth analysis of this matrix structure, we uncover a Sudoku adjacency algebra. This algebraic framework is constructed from a coherent configuration, comprised of equivalence relations among row-column, row-symbol, column-symbol, and box-symbol Sudoku conditions.

The result we present is a degree threshold for G_P that allows a fractional tile-decomposition, implying the existence of a fractional completion of P . The proof employs spectral decomposition, the properties of coherent configurations, and perturbation theory to estimate a generalized inverse for the matrix representation of a partial Sudoku puzzle in order to find a solution for the relaxed linear system. Improving on this result by finding a minimum degree threshold for an exact tile-decomposition is an interesting open question in this research area.