DANIEL DALLAIRE, University of Ottawa

The Chromatic Category: A Connection Between Planar Graph Colouring And Representation Theory

The chromatic category is a diagrammatic monoidal category which encodes information about the colourings of planar graphs. Although it has a direct relation to graph theory in this way, it can also be constructed in a representation theoretic context yielding an interesting connection between the problem of colouring planar graphs (in particular, the four colour theorem) and representation theory. In this work I explore these connections and prove results about the chromatic category. One main result is that this category is isomorphic to a category of representations of quantum $\mathfrak{sl}_2$, which in the 4-colour case, tells us the category is equivalent to a category of representations of (non-quantum) $\mathfrak{sl}_2$. In addition to this, I give bases for the morphism spaces in the category, discuss the category's relation to the well-known Temperley-Lieb category, and show that it is pivotal (allowing for topological arguments on the morphisms). Lastly, I give different presentations of the category, all of which are isomorphic to the chromatic category.