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On Quantum Circuits Enacting the E8 Weyl Group

The circuit diagrams studied in computer science enjoy a rich mathematical theory. Starting from a finite set of primitive operators known as "gates", a circuit diagram is any operator obtained by composing a finite number of gates in sequence or in parallel. A special class of circuit diagrams are the classical reversible circuits, in which gates are invertible matrices over $\mathbb{Z}_2$. It was shown by Toffoli in 1980 that every classical reversible circuit is constructible from a single primitive known as the Toffoli gate, given sufficiently many ancillas (i.e., working memory). Later, Feynman generalized classical reversible circuits to quantum mechanical systems in which invertible matrices are replaced by unitary matrices. Since unitary matrices are uncountable, there does not exist an exact universal gate set for quantum computation. However, given both the Toffoli gate and Hadamard gate, all unitary operators can be simulated.

In this poster, we describe ongoing work to obtain a minimal presentation for the group G of 3-qubit ancilla-free Toffoli+K circuits (where K is the two-fold tense of a Hadamard gate). From prior results in sphere packings, it follows that G is isomorphic to the E8 Weyl group. We start from the Coxeter presentation of this Weyl group and obtain a circuit presentation using a semantic variation of Tietze transformations. We use commutativity relations to prove generator minimality and implement a proof-assistant to validate each semantic transformation. Directions for future work, such as minimizing the set of relations and generalizing to the 3-qubit ancilla-free Toffoli+K circuits, are outlined.