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Sharp convergence rates for mean field control on the region of strong regularity

This talk will be about the convergence of certain symmetric N-particle stochastic control problems towards their mean field limits. After a brief introduction to mean field control, we will mainly discuss the following question: how fast do the value functions V^N for the N-particle problems converge towards the value function U of the mean field problem? Or in terms of partial differential equations - how fast do the solutions of certain finite-dimensional Hamilton-Jacobi-Bellman equations converge to the solution of a corresponding Hamilton-Jacobi-Bellman equation set on the space of probability measures? If the data is smooth and convex, then U is smooth, and the rate is O(1/N). When the data is not convex, U may fail to be smooth, and the answer is more subtle. On one hand, we know that the optimal global rate cannot be better than $O(1/\sqrt{N})$. On the other hand, a recent paper of Cardaliaguet and Souganidis identifies an open and dense set \mathcal{O} of initial conditions (which we call the region of strong regularity, by analogy with some classical results on first order Hamilton-Jacobi equations) where Uis smooth, and it is natural to wonder whether the rate of convergence might be better inside of \mathcal{O} . In an ongoing joint work with Cardaliaguet, Mimikos-Stamatopoulos, and Souganidis, we show that this is indeed the case: the rate is O(1/N) locally uniformly inside the set \mathcal{O} , so the convergence is indeed faster inside \mathcal{O} than it is outside.