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Multi-Level Approach for Error Reduction in Physics-Informed Neural Networks

In recent years, deep learning approaches, such as the physics-informed neural networks (PINNs), have shown promising results for several classes of initial and boundary-value problems. However, their ability to surpass, particularly in terms of accuracy, classical discretization methods such as the finite element methods, remains a significant challenge. One of the main obstacles of deep learning approaches lies in their inability to consistently reduce the relative error in the computed solution. We present our novel approach, the multi-level neural networks, in order to reduce the solution error when using deep learning approaches. The main idea consists in computing an initial approximation to the problem using a simple neural network and in estimating, in an iterative manner, a correction by solving the problem for the residual error with a new network of increasing complexity. This sequential reduction of the residual associated with the partial differential equation allows one to decrease the solution error, which, in some cases, can be reduced to machine precision. The underlying explanation is that the method is able to capture at each level smaller scales of the solution using a new network. Numerical examples in 1D and 2D dealing with linear and non-linear problems are presented to demonstrate the effectiveness of the proposed approach using PINNs.