

---

**MARK IWEN**, Michigan State University

*Sparse Spectral Methods for Solving High-Dimensional and Multiscale Elliptic PDEs*

In his monograph "Chebyshev and Fourier Spectral Methods", John Boyd claimed that, regarding Fourier spectral methods for solving differential equations, "[t]he virtues of the Fast Fourier Transform will continue to improve as the relentless march to larger and larger [bandwidths] continues". This talk will discuss attempts to further the virtue of the Fast Fourier Transform (FFT) as not only bandwidth is pushed to its limits, but also the dimension of the problem. Instead of using the traditional FFT however, we make a key substitution from the sublinear-time compressive sensing literature: a high-dimensional, sparse Fourier transform (SFT) paired with randomized rank-1 lattice methods. The resulting sparse spectral method rapidly and automatically determines a set of Fourier basis functions whose span is guaranteed to contain an accurate approximation of the solution of a given elliptic PDE. This much smaller, near-optimal Fourier basis is then used to efficiently solve the given PDE in a runtime which only depends on the PDE's data/solution compressibility and ellipticity properties, while breaking the curse of dimensionality and relieving linear dependence on any multiscale structure in the original problem. Theoretical performance of the method is established with convergence analysis in the Sobolev norm for a general class of nonconstant diffusion equations, as well as pointers to technical extensions of the convergence analysis to more general advection-diffusion-reaction equations. Numerical experiments demonstrate good empirical performance on several multiscale and high-dimensional example problems, further showcasing the promise of the proposed methods in practice.