Descriptive Set Theory

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RACHAEL ALVIR, University of Waterloo
Scott Complexity

The logic $L_{\omega_1\omega}$ is the extension of finitary first-order logic allowing countably infinite conjunctions and disjunctions. In this logic, every countable structure can be characterized up to isomorphism (among countable structures) by a single sentence known as a Scott sentence. The proof of this result reveals that a special ordinal, known as the Scott Rank of the structure, is of interest. Unfortunately, many non-equivalent definitions of Scott rank exist in the literature. In an attempt to standardize the definition of Scott rank, Antonio Montalban argued that we should define the Scott rank of A to be the least α such that A has a $\Pi_{\alpha+1}$ Scott sentence. This notion of Scott rank is robust, having many equivalent characterizations. In particular, one is able to show that this condition is equivalent to the set of presentations of A being boldface $\Pi_{\alpha+1}$ in the Borel hierarchy in the space Mod(L). However, when A has a Scott sentence with a different complexity also has many equivalent characterizations. The notion of Scott complexity is arguably the most refined such notion which retains a connection with the Borel hierarchy. In this talk, we introduce Scott complexity and then compute the Scott complexity for several classes of structures.

SUMUN IYER, Cornell University
Generic homeomorphisms of Knaster continua

Knaster continua are a class of compact, connected, metrizable spaces which are indecomposable in the sense that they cannot be written as the union of two proper non-trivial compact, connected subspaces. Let K be the universal Knaster continuum (this is a unique Knaster continuum which continuously and openly surjects onto all other Knaster continua). The group $\mathrm{Homeo}(K)$ of all homeomorphisms of the universal Knaster continuum is a non-locally compact Polish group. We prove that it contains an open, normal subgroup which has a comeager conjugacy class.

CHRIS KARPINSKI, McGill University
Hyperfiniteness of boundary actions of groups

Groups with some notion of negative curvature (such as free groups, and more generally hyperbolic groups and their various generalizations) have a notion of a boundary at infinity, which is a Polish space on which the group acts by homeomorphisms. The actions of such groups on their boundaries have been shown to furnish examples of hyperfinite orbit equivalence relations, and hence have been of interest in descriptive set theory. We survey results on hyperfiniteness of boundary actions of various "negatively curved" groups, beginning with the simple case of free groups and demonstrating how the methods for free groups are applied to more general groups. We show, however, that these methods break down at the level of generality of "acylindrically hyperbolic groups", by outlining a construction of an acylindrically hyperbolic group exhibiting a non-hyperfinite boundary action.

KOICHI OYAKAWA, Vanderbilt University
Hyperfiniteness of boundary actions of acylindrically hyperbolic groups

A Borel equivalence relation on a Polish space is called hyperfinite if it can be approximated by equivalence relations with finite classes. This notion has long been studied in descriptive set theory to measure complexity of Borel equivalence relations. Although group actions on hyperbolic spaces don't always induce hyperfinite orbit equivalence relations on the Gromov boundary, some natural boundary actions were recently found to be hyperfinite. Examples of such actions include actions of hyperbolic groups and relatively hyperbolic groups on their Gromov boundary, actions of mapping class groups on arc graphs and curve graphs, and acylindrical group actions on trees. In this talk, I will show that any acylindrically hyperbolic group admits a non-elementary acylindrical action on a hyperbolic space with hyperfinite boundary action.

ANTOINE POULIN, McGill
Space of Archimedean Left-Orders

We prove that the space of Archimedean orders on $\mathbb{Z}^3$ with the equivalence relation induced by the action of $\mathrm{GL}(n, \mathbb{Z})$ is not hyperfinite.

FORTE SHINKO, University of California, Berkeley
Equivalence relations classifiable by Polish abelian groups

A conjecture of Hjorth states that the only countable Borel equivalence relations reducible to orbit equivalence relations of Polish abelian group actions are the hyperfinite ones. This conjecture was recently refuted by Allison, where he showed that every treeable countable Borel equivalence relation reduces to such an orbit equivalence relation. We show that this holds for more general equivalence relations, including all countable Borel equivalence relations. This is joint with Josh Frisch.

IIAN SMYTHE, University of Winnipeg
A descriptive approach to manifold classification

We propose a unified descriptive set-theoretic framework for studying the complexity of classification problems arising in geometric topology. We establish several precise complexity results, such as for the classification of surfaces up to homeomorphism, and for classes of hyperbolic manifolds up to isometry. The latter is intimately connected with the conjugation actions of certain Lie groups on their spaces of discrete subgroups. This work is joint with Jeffrey Bergfalk.

SPENCER UNGER, University of Toronto
Circle squaring with algebraic irrational translations

I will describe some joint work with Andrew Marks where we show that known circle squaring results can be done with translations whose coordinates are algebraic irrational.

JENNA ZOMBACK, University of Maryland, College Park
Boundary actions of free semigroups

We consider the natural action of a free, finitely generated semigroup (the set of all finite words in a finite alphabet S) on its boundary (the space of infinite words in S) by concatenation. While boundary actions of free groups are well-studied, much less is known for semigroups. In joint work with Anush Tserunyan, we completely characterize those Markov measures which make the boundary action weakly mixing (i.e., the product with an ergodic probability measure preserving action is ergodic). This is an ingredient in the proof of pointwise ergodic theorems for measure preserving actions of free semigroups.