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Generic properties of eigenfunctions in the presence of torus actions

Let G be a compact Lie group acting on a closed manifold M. Inspired by work of Uhlenbeck (1976), we explore the generic properties of Laplace eigenfunctions associated to G-invariant metrics on M. We find that, in the case where $\mathbb{T}$ is a torus acting freely on M, the Laplace eigenspaces associated to a generic $\mathbb{T}$-invariant metric are irreducible representations of $\mathbb{T}$. This provides a mathematically rigorous instance of the belief in quantum mechanics that, in the presence of symmetry, non-irreducible eigenspaces are "accidental degeneracies." Turning to nodal sets of Laplace eigenfunctions, we find that for a generic $\mathbb{T}$-invariant metric on M the nodal sets are embedded hypersurfaces. Additionally, under suitable conditions, our framework allows us to prove the existence of numerous examples of Riemannian manifolds for which, modulo a subspace of Weyl density zero, every non-constant Laplace eigenfunction has precisely two nodal domains, which is the minimal possible number.

This is joint work with Donato Cianci (GEICO), Chris Judge (Indiana) and Samuel Lin (Oklahoma).