Cluster Algebras in Representation Theory
Algèbres amassées dans la théorie des representations

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TOM GANNON, UCLA
Proof of the Ginzburg-Kazhdan conjecture

The main theorem of this talk will be that the affine closure of the cotangent bundle of the basic affine space (also known as the universal hyperkahler implosion) has symplectic singularities for any reductive group, where essentially all of these terms will be defined in the course of the talk. After discussing some motivation for the theory of symplectic singularities, we will survey some of the basic facts that are known about the universal hyperkahler implosion and discuss how they are used to prove the main theorem. Time permitting, we will also discuss recent work in progress, joint with Harold Williams, which identifies the universal hyperkahler implosion in type A with a Coulomb branch in the sense of Braverman, Finkelberg, and Nakajima, confirming a conjectural description of Dancer, Hanany, and Kirwan.

JAMES HUGHES, Duke University
Cluster Modular Groups of Braid Varieties

Braid varieties are a family of cluster varieties that arise naturally from the study of Legendrian links and their exact Lagrangian fillings. They encompass several known families of cluster varieties, including positroid varieties, double Bott-Samelson varieties, and open Richardson varieties. The symplectic-geometric origins of braid varieties lead to a combinatorial interpretation of clusters as weaves – colored graphs satisfying certain properties. In this talk I will discuss how to use these combinatorics to describe cluster modular groups in various known and conjectural cases, including in the case of Grassmannians and some of their foldings.

JOEL KAMNITZER, McGill University
Cluster algebras, MV polytopes, and MV cycles

I will present some combinatorial observations concerning the relationship between the cluster algebra structure on the coordinate ring of unipotent subgroup and Mirkovic-Vilonen polytopes. Then I will give some speculation on the relationship between this cluster algebra and the theory of MV cycles.

ALEXIS LEROUX LAPIERRE, McGill
An algebraic equivariant multiplicity using limits of characters

The seemingly elementary question of writing down perfect bases for the irreducible representations of semisimple Lie algebras is a problem which finds its source in surprisingly involved mathematical tools. Two such sources are a version of the geometric Satake equivalence (giving rise to the so-called Mirkovic-Vilonen bases) and a categorification of U_q^- using KLR algebras (giving rise to the so-called dual canonical bases). It has been shown that these two families of bases do not coincide, raising the question of understanding the change of basis matrix. To bridge these two different constructions, we introduce a new notion of an algebraic equivariant multiplicity for modules over truncated shifted Yangians through limits of characters. We relate it to some well-studied functions on modules over KLR algebras and to the usual notion of equivariant multiplicity of MV cycles. This is joint work with Anne Dranowski and Joel Kamnitzer.

YU LI, University of Toronto
Integrable systems on the dual of nilpotent Lie subalgebras and T-Poisson cluster structures

Let $\mathfrak{g}$ be a semisimple Lie algebra and $\mathfrak{g} = \mathfrak{n} \oplus \mathfrak{h} \oplus \mathfrak{n}_-$ a triangular decomposition. Motivated by a construction of Kostant-Lipsman-Wolf, we construct an integrable system on the dual space of $\mathfrak{n}_-$ equipped with the Kirillov-Kostant Poisson structure. The Bott-Samelson coordinates on the open Bruhat cell (equipped with the standard Poisson structure) makes it into a symmetric Poisson CGL extension, hence giving rise to a T-Poisson cluster structure on it. Our integrable system is obtained from the initial cluster by taking the lowest degree terms of the initial cluster variables. We conjecture that mutation of clusters gives rise to mutation of integrable systems. This is joint work in progress with Yanpeng Li and Jiang-Hua Lu.

DINUSHI MUNASINGHE, University of Toronto
Schur Algebras in Type B

In type A, the q-Schur algebra of Dipper and James forms a graded cellular quasi-hereditary cover of the Hecke algebra as the commutant via Jimbo's quantum Schur–Weyl duality. In type B, however, the commutant $\mathcal{L}^n(m)$ and the quasi-hereditary cover $\mathcal{S}^n(\Lambda)$ (the cyclotomic q-Schur algebra of Dipper, James and Mathas) are non-isomorphic. At generic parameters they are both Morita equivalent to the type B Hecke algebra, but this fails at special parameters. By realizing $\mathcal{L}^n(m)$ as an idempotent truncation of $\mathcal{S}^n(\Lambda)$ we leverage the well-known structure of the cyclotomic q-Schur algebra to investigate the representation theory of $\mathcal{L}^n(m)$.

THEO PINET, Université Paris Cité and Université de Montréal
Inflations for representations of shifted quantum affine algebras

It is well-known that the only finite-dimensional simple Lie algebra admitting a 2-dimensional irreducible representation is $\mathfrak{sl}_2$. The restriction functors appearing in classical Lie theory are therefore not dense on simple modules. The goal of this talk is to show that this density property is however satisfied in the setting of shifted quantum affine algebras (SQAs for short).

SQAs are infinite-dimensional algebras parametrized by a finite-dimensional simple Lie algebra $\mathfrak{g}$ and a coweight of this Lie algebra. They are of fundamental importance in the modern formulation of representation theory and play an essential role in geometry, in quantum integrable systems and in the study of cluster algebras. Let us fix a SQA and denote it by U. Then, like in classical Lie theory, for any choice of simple root of the Lie algebra $\mathfrak{g}$ underlying U, there is a corresponding subalgebra U' which is isomorphic to a SQA of rank 1 (i.e. whose underlying Lie algebra is $\mathfrak{sl}_2$). A natural question to ask is thus whether or not any simple representation of this subalgebra can be lifted to a simple representation of U. The answer is yes and we can even choose these "lifts" so that the action of another important subalgebra of U (which is almost complementary to U' in some sense) is trivial. These special "lifts" are called *inflations*.

The main result of this talk will be an existence theorem for inflations of representations of rank 1 SQAs. We will also present several potential applications of this theorem.

HUGH THOMAS, UQAM
Generalized associahedra as moment polytopes

Generalized associahedra are a well-studied family of polytopes associated to a finite-type cluster algebra and choice of starting cluster. We show that the generalized associahedra constructed by Padrol, Palu, Pilaud, and Plamondon, building on ideas from Arkani-Hamed, Bai, He, and Yan, can be naturally viewed as moment polytopes for an open patch of the quotient of the $\mathcal{A}$-cluster variety with universal coefficients by its maximal natural torus action. We prove our result by showing that the construction of Padrol, Palu, Pilaud, and Plamondon can be understood on the basis of the way that moment polytopes behave under symplectic reduction.

This is joint work with Michael Gekhtman.

GORDANA TODOROV, Northeastern University
Higher Auslander Algebras and Fundamental Domains of Cluster Categories

Endomorphism algebras of the fundamental domains of acyclic m-cluster categories (viewed as full subcategories of the appropriate derived categories) are shown by Emre Sen to be of higher representation finite type in the sense of Iyama. Iyama introduced the notion of higher Auslander algebras and higher representation finite algebras and he described the relation between those, generalizing the well known result of Auslander about the correspondence between Auslander algebras and algebras of finite representation type, up to Morita equivalence.

Auslander algebras are defined as (global dimension A $\leq 2 \leq$ dominant dimension A) and higher k-Auslander algebras are defined as (global dimension A $\leq$ k + $1 \leq$ dominant dimension A).

KAYLA WRIGHT, University of Minnesota
Higher Dimers, Webs and Grassmannian Cluster Algebras

In this talk, we will discuss joint work with Moriah Elkin and Gregg Musiker about a combinatorial model for certain Grassmannian cluster algebras. The Grassmannian of k-planes in $\mathbb{C}^n$, Gr(k,n), has a cluster structure that is not well-understood for $k > 2$. In these algebras, Plücker coordinates Δ_I give us a subset of the cluster variables and have lovely combinatorial descriptions. However, most cluster variables are more complicated expressions in Plücker coordinates and lack such a combinatorial description. In our work, we give a graph theoretic interpretation for the Laurent expansion of cluster variables of low degree in terms of higher dimer models. This work employs SL_k-web combinatorics and we conjecture these webs are the key ingredient to understanding Grassmannian cluster algebras. If time permits, I would like to also pose an open problem I hope to work on relating our dimer combinatorics to the categorification of Grassmannian cluster algebras.

MILEN YAKIMOV, Northeastern University
Finite generation and representation theory of quantum cluster algebras at roots of unity

We will address two problems on quantum cluster algebras. The first is about transferring finite generation from classical to quantum cluster algebras and back. We will describe an if and only if result, based on techniques from Cayley-Hamilton algebras. The second problem is about the classification of irreducible representations of quantum cluster algebras at roots of unity. We will describe those of maximal dimension, i.e., the so called Azumaya loci. The talk is based on joint works with Shengnan Huang, Thang Le, Greg Muller, Bach Nguyen and Kurt Trampel.

KAREN YEATS, University of Waterloo
T-duality by Le diagrams

Lukowiski, Parisi, and Williams gave a combinatorial version of the T-duality map of string theory in terms of decorated permutations. I will explain how to see this map at the level of Le diagrams, a perspective which makes the dimension relation more transparent. The talk will be combinatorial, and while no cluster algebras will appear directly, hopefully the audience will find interesting connections and synergies. Joint work with Simone Hu.

YAN ZHOU, Northeastern