The standard lens cluster in \( \mathbb{R}^2 \) uniquely minimizes relative perimeter

We present a new type of planar partitioning problem in which one minimizes perimeter among clusters with one domain of finite area and two of infinite area. This generalizes the classical setting where one minimizes perimeter among clusters with a fixed number of domains of finite prescribed area. The classical isoperimetric problems have led to the famous double and triple bubble conjectures, which have both been proven to hold in the Euclidean plane. We use the planar double bubble theorem to show that the lens cluster is the unique minimizer for the isoperimetric problem of partitioning the plane into three disjoint domains, one having unit area and the remaining two having infinite area. In addition to the result, we present a general framework and several conjectures for this new class of problems in geometric measure theory.