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Free extreme points of free spectrahedrops and generalized free spectrahedra

Matrix convexity generalizes convexity to the dimension free setting and has connections to many mathematical and applied pursuits including operator theory, quantum information, noncommutative optimization, and linear control systems. In the setting of classical convex sets, extreme points are central objects. For example, the well-known Minkowski theorem shows that any element of a closed bounded convex set can be expressed as a convex combination of extreme points. Extreme points are also of great interest in the dimension free setting of matrix convex sets; however, here the situation requires more nuance, as there are many types of extreme points for matrix convex sets. Of particular interest are free extreme points, a highly restricted type of extreme point that is connected to the dilation theoretic Arveson boundary.

Building on a recent work of J. W. Helton and the speaker which shows that free spectrahedra, i.e., dimension free solution sets to linear matrix inequalities, are spanned by their free extreme points, this talk establishes two additional classes of matrix convex sets that are spanned by their free extreme points. Namely, we show that closed bounded free spectrahedrops, i.e, closed bounded projections of free spectrahedra, are the span of their free extreme points. Furthermore, we show that if one considers linear operator inequalities that have compact operator defining tuples, then the resulting "generalized" free spectrahedra are spanned by their free extreme points.