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Integers with a sum of co-divisors yielding a square

Finding elliptic curves with high ranks has been the focus of much research. Recently, with the goal of generating elliptic curves with a large rank, some authors used large integers n which have many divisors, amongst which one can find divisors d such that $d+n/d$ is a perfect square. This strategy is in itself a motivation for studying the function $\tau_\square(n)$ which counts the number of divisors d of an integer n for which $d+n/d$ is a perfect square. We show that $\sum_{n\le x}\tau_\square(n)=c_\square x^{3/4}+O(\sqrt{x})$ for some explicit constant $c_\square$. Moreover, letting $\rho_1(n):=\max\{d\mid n: d\le\sqrt{n}\}$ and $\rho_2(n):=\min\{d\mid n: d\ge\sqrt{n}\}$ stand for the middle divisors of n, we show that the order of magnitude of the number of positive integers $n\le x$ for which $\rho_1(n)+\rho_2(n)$ is a perfect square is $x^{3/4}/\log x$. This is joint work with Jean-Marie De Koninck and Hans Schmidt Ramiliarimanana.