
Algebraic, Arithmetic and Kahler Geometry: Recent developments

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HOUARI BENAMMAR AMMAR, UQAM

Slope inequality for an arbitrary divisor.

In this talk, we explain a more general version of slope inequality for a fibred surface, we compute some natural examples, and we give applications. Our approach applies the theory of Harder-Narasimhan filtrations for vector bundles on curves and builds on the approach of G. Xiao.

DAVE ANDERSON, Ohio State University

Refined transversality and equivariant positivity

The standard Kleiman-Bertini transversality theorems say that if a variety is homogeneous with respect to the action of an algebraic group, this action moves any two subvarieties into transverse position. I will describe refinements which treat cases where the action is not transitive, along with an application to the positivity of cohomology and K-theory classes of subvarieties of a generalized flag variety.

MICHAEL GROECHENIG, University of Toronto

p-adic integration, buildings and BPS invariants

I will report on joint work in progress with Dimitri Wyss and Paul Ziegler, continuing our previous papers on the Hausel-Thaddeus conjecture and Ngô's geometric stabilisation theorem. In those articles, we applied Batyrev's technique of p-adic integration to the Hitchin system. A common feature of our previous two papers is that we worked with moduli spaces, which have at most quotient singularities. We achieved this either by imposing a coprimality assumption on rank and degree, or by restricting to the elliptic locus. Our new work sheds light on the geometric meaning of p-adic integration in the absence of those assumptions. We will see that the resulting theory is closely related to BPS invariants.

CHANGHO HAN, University of Waterloo

Extending Torelli map from Smyth's alternative compactifications of the moduli of curves

It is well-known that the Torelli map, that turns a smooth curve of genus g into its Jacobian (a principally polarized abelian variety of dimension g), extends to a map from the Deligne—Mumford moduli of stable curves to the moduli of semi-abelic varieties by Alexeev. Moreover, it is also known that the Torelli map does not extend over the alternative compactifications of the moduli of curves as described by the Hassett—Keel program, including the moduli of pseudostable curves (can have nodes and cusps but not elliptic tails). But it is not yet known whether the Torelli map extends over the Smyth's alternative compactifications of the moduli of curves; what about the moduli of curves of genus g with rational m -fold singularities, where m is a positive integer bounded above? As a joint work in progress with Jesse Kass and Matthew Satriano, I will describe two types of moduli of curves with m -fold singularities and describe how far the Torelli map extends over such spaces into the Alexeev compactifications.

ANDREW HARDER, Lehigh University

Tropical homology and mirror symmetry

Tropical homology, introduced by Mikhalkin and Zharkov, gives a powerful combinatorial tool for describing Hodge numbers of projective varieties. In this talk, I will explain how one can use tropical homology to prove a conjecture of Katzarkov, Kontsevich, and Pantev for toric complete intersections. This is joint work in progress with Sukjoo Lee.

KATRINA HONIGS, Simon Fraser University

Hyperkahler varieties of Kummer type and torsion points of abelian surfaces

In this talk, I will describe the geometry of part of the middle cohomology of some hyperkahler varieties of Kummer type and the Galois action on it. I will then discuss how these results can be applied to find abelian surfaces whose generalized Kummer 4-folds are not derived equivalent, over a ground field of \mathbb{Q} , to the generalized Kummer 4-folds of their duals.

JOEL KAMNITZER, McGill University

Moduli space of cactus flower curves

I will discuss a variant on the Deligne Mumford space of genus 0 curves which parametrizes "cactus flower curves". This moduli space is constructed starting with the moduli space of flower curves, which is the matroid Schubert variety associated to the braid arrangement. The fundamental group of the real locus of this space is the virtual cactus group.

JULIEN KELLER, UQAM

Variational and non-archimedean aspects of the correspondence for vector bundles

The famous correspondence for vector bundles, proved by Donaldson and Uhlenbeck-Yau states that the existence of Hermitian-Einstein metrics on a holomorphic vector bundle is equivalent to an algebro-geometric stability condition. Using a variational formulation, we will explain several results that provide a link between differential geometry and algebraic geometry in the above correspondence. Our approach is based on the study of an object called the Quot-scheme limit of Fubini-Study metrics, which is used to evaluate certain algebraic 1-parameter subgroups of Hermitian metrics by using the notion of Quot-schemes introduced by Grothendieck. We will present a dictionary between non-archimedean aspects related to the correspondence for bundles and non-archimedean aspects related to the Yau-Tian-Donaldson conjecture for varieties. This a joint work with Y. Hashimoto (Osaka).

STEVEN LU, UQAM

Rigidity of maps into moduli space of polarized varieties

I will present the the problem and the methodology of the solution of the rigidity of pointed maps from an arbitrary algebraic variety to the moduli space of polarized (smooth) good minimal models. This is joint work with Ariyan Javanpaykar, Ruiran Sun and Kang Zuo.

SAM PAYNE, University of Texas at Austin

Cohomology groups of moduli spaces of curves

The cohomology groups of moduli spaces of curves are important to several mathematical disciplines, from low-dimensional topology and geometric group theory to stable homotopy theory and quantum algebra. Algebraic geometry endows these groups with additional structures, such as Hodge structures and Galois representations, and the Langlands program makes striking predictions about which such structures can appear. I will survey recent results confirming several of these predictions and making progress toward calculating these groups and determining in which degrees they do and do not vanish.

DEBADITYA RAYCHAUDHURY, University of Arizona

On the singularities of secant varieties

In this talk, we study the singularities of secant varieties of smooth projective varieties when the embedding line bundle is sufficiently positive. We give a necessary and sufficient condition for these to have p-Du Bois singularities. In addition, we show that the singularities of these varieties are never higher rational. From our results, we deduce several consequences, including a

Kodaira-Akizuki-Nakano type vanishing result for the reflexive differential forms of the secant varieties. Work in collaboration with S. Olano and L. Song.

ZINOVY REICHSTEIN, Department of Mathematics, University of British Columbia
Essential dimension of symmetric groups in prime characteristic

Computing the essential dimension of the symmetric group S_n is a long-standing open problem, originating in the work of Felix Klein. It is known that this number lies between $\lfloor n/2 \rfloor$ and $n - 3$ for any $n \geq 5$. The exact value is not known for any $n \geq 8$, though it is expected to be $n - 3$ for every $n \geq 5$, at least in characteristic 0. The main result of this talk, based on joint work with Oakley Edens, is that for odd prime p there are infinitely many positive integers n such that the essential dimension of S_n is $\leq n - 4$ in characteristic p .

ETHAN ROSS, University of Toronto
Singular Reduction of Polarizations

A polarization on a symplectic manifold (M, ω) is an involutive complex Lagrangian subbundle P of the complexified tangent bundle $T^{\mathbb{C}}M$. Kähler structures are special cases of polarizations which intersect their complex conjugates trivially. Much work has been done discussing how Kähler structures behave under symplectic reduction, with only partial results for the reduction of more general polarizations. In this talk I will discuss the reduction of polarizations and also extend to the setting of singular reduction explored by Sjamaar-Lerman.

CARLO SCARPA, UQAM
The Einstein-Hilbert functional and K-stability

Given a polarised manifold $L \rightarrow X$, we explain how K-stability of (X, L) is related to properties of (a version of) the Einstein-Hilbert functional of the circle bundle associated to $L \rightarrow X$. This strongly hints at a possible connection between the Yau-Tian-Donaldson conjecture and the Yamabe problem; to exemplify how this point of view can be useful to understand the geometry of polarised manifolds, we will show a new proof of K-semistability of polarised manifolds admitting constant scalar curvature Kähler metrics. Based on arXiv:2310.11625, joint work with Abdellah Lahdili and Eveline Legendre.

SASHA ZOTINE, Queen's University
Kawaguchi-Silverman Conjecture for Projective Bundles on Curves

The Kawaguchi-Silverman Conjecture (KSC) is a recent conjecture equating two invariants of a dominant rational map between projective varieties: the first dynamical degree and arithmetic degree. The first dynamical degree measures the topological mixing of the map, and the arithmetic degree measures how complicated rational points become after iteration. Recently, the conjecture was established for several classes of varieties, including projective bundles over any non-elliptic curve. Together with Brett Nasserden (Western Ontario), I resolve the elliptic case, hence proving KSC for all projective bundles over curves.