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Admissibility is Bayes optimality with infinitesimals

In this talk, I'll summarize recent work exploiting tools in mathematical logic to resolve longstanding open problems in statistical decision theory. I'll focus on an exact characterization of admissibility in terms of Bayes optimality in the nonstandard extension of the original decision problem, as introduced by Duanmu and Roy (Ann. Statist. 49(4): DOI:10.1214/20-AOS2026). Unlike the consideration of improper priors or other generalized notions of Bayes optimality, the nonstandard extension is distinguished, in part, by having priors that can assign "infinitesimal" mass in a sense that is made rigorous using results from nonstandard analysis. With these additional priors, we find that, informally speaking, a decision procedure δ_0 is admissible in the original statistical decision problem if and only if, in the nonstandard extension, the nonstandard extension of δ_0 is Bayes optimal among the extensions of standard decision procedures with respect to a nonstandard prior assigning at least infinitesimal mass to every standard parameter value. We use this theorem to give further characterizations of admissibility, one related to Blyth's method and another related to a condition due to Stein that characterizes admissibility under regularity. Our results imply that Blyth's method is a sound and complete method for establishing admissibility. Buoyed by this result, we revisit the univariate two-sample common-mean problem, and show that the Graybill–Deal estimator is admissible among a certain class of unbiased decision procedures.

Joint work with Haosui Duanmu and David Schrittesser.