Recent advances on nonlinear evolution equations
Développements récents dans le domaine des équations d'évolution non linéaires
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GAEL YOMGNE DIEBOU, The Fields Institute for Research in Mathematical Sciences
Remarks on the heat flow of harmonic maps: uniqueness and weak-strong theory

There are essentially two research lines pertaining to the harmonic maps problem: the first is related to the theory of weak solutions whose global existence is proved for initial data in $W^{1,2}$ and the second to the theory of strong (mild) solutions which are constructed in scaling invariant spaces. The uniqueness question for both notion of solutions is largely open. In this talk, I will discuss a new uniqueness result and its consequence in bridging the gap between finite energy weak solutions and mild solutions.

MANUELA GIROTTI, Saint Mary's University
The dynamics soliton gasses: Fredholm determinants, asymptotics, and kinetic equations

I will describe a collection of collaborations with K. McLaughlin (Tulane U.), T. Grava (SISSA/Bristol), R. Jenkins (UCF) and A. Minakov (U. Karlova). We analyze the case of a regular, dense (modified) KdV soliton gas and its large time behaviour in the presence of a single trial soliton. We show that the solution can be decomposed as the sum of the background gas solution (a modulated elliptic wave), plus a soliton solution: the individual expressions are however quite convoluted due to the interaction dynamics. We are able to derive the local phase shift of the gas after the passage of the soliton, and we can trace the location of the soliton peak as the dynamics evolves; additionally, we show that the soliton peak, while interacting with the soliton gas, has an oscillatory velocity whose leading order average value satisfies the kinetic velocity equation analogous to the one posited by V. Zakharov and G. El.

SLIM IBRAHIM, University of Victoria

ADILBEK KAIRZHAN, University of Toronto
Asymptotic stability near the soliton for quartic Klein-Gordon in 1D

In this talk we discuss the nonlinear focusing Klein-Gordon equation in $1+1$ dimensions and the global space-time dynamics of solutions near the unstable soliton. We give a proof of optimal decay, and local decay, for even perturbations of the static soliton originating from well-prepared initial data belonging to a subset of the stable manifold constructed in Bates-Jones (Dynamics reported, 1989) and Kowalczyk-Martel-Muñoz (J. Eur. Math. Soc., 2021). Our results complement those of Kowalczyk-Martel-Muñoz (J. Eur. Math. Soc., 2021) and confirm numerical results of Bizon-Chmaj-Szpak (J. Math. Phys., 2011). In particular, we provide new information both local and global in space about asymptotically stable perturbations of the soliton under localization assumptions on the data.

This is a joint work with Fabio Pusateri.

GIUSY MAZZONE, Queen's University
Periodic motion of a harmonic oscillator interacting with a viscous fluid

We consider the motion of a harmonic oscillator immersed in a viscous incompressible fluid within an infinite pipe. The motion of the fluid is driven by a prescribed, time-periodic flow rate. As the fluid flows in the channel, it may exert a periodic force on the oscillator. In this setting, if the frequency of this force matches the natural frequency of the oscillator, then the phenomenon of resonance may occur with the mass oscillating with increasing amplitude. Because of the phenomenon of resonance, the motion of the harmonic oscillator would not be time-periodic. We will show that resonance does not occur in the class of weak solutions to the governing equations if the flow rate is "sufficiently small". In addition, we will prove that -at a large distance from the oscillator- the fluid velocity converges to the time-periodic generalization of the Poiseuille flow in an infinite pipe.

MANUEL PALACIOS, University of Toronto
Asymptotic Stability of Peakons for the Novikov equation

The Novikov equation is an integrable Camassa-Holm-type equation with a cubic nonlinearity. One of its most important features is the existence of peaked traveling waves. In this talk, we will prove the asymptotic stability of those peakon solutions, under $H^1(\mathbb{R})$-perturbations satisfying that their associated momentum density defines a non-negative Radon measure. In order to do that, we first prove a rigidity theorem, sometimes called Liouville theorem. The main novelty in our analysis, compared to the Camassa-Holm case, comes from the fact that the momentum is not a conserved quantity anymore. To overcome this problem, we introduce a new Lyapunov functional unrelated to the (non-conserved) momentum of the equation.

JIA SHI, MIT
On the analyticity of the Muskat equation

The Muskat equation describes the interface of two liquids in a porous medium. We will show that if a solution to the Muskat problem in the case of same viscosity and different densities is sufficiently smooth, then it must be analytic except at the points where a turnover of the fluids happens. We will also show analyticity in a region that degenerates at the turnover points provided some additional conditions are satisfied.

YAKOV SHLAPENTOKH-ROTHMAN, University of Toronto
Self-Similarity for the Einstein Vacuum Equations and Applications

I will discuss old and new notions of self-similarity for the Einstein Vacuum Equations and discuss applications of these such as the construction of Naked Singularities.

MICHAEL SIGAL, University of Toronto
Vacuum solutions of the theory of electroweak interactions

In this talk I will describe the vacuum sector of the Weinberg-Salam (WS) model of electroweak forces. (In the vacuum sector the WS model yields the $U(2)$-Yang-Mills-Higgs equations.) We show that at large constant magnetic fields the translational symmetry of the equation is broken spontaneously: the solutions, with the lowest energy locally, in the plane orthogonal to the magnetic field, have the symmetry of a lattice. The stability of these solutions is an open problem.

CATHERINE SULEM, University of Toronto
A Hamiltonian approach to nonlinear modulation of surface water waves in the presence of linear shear currents.

This is a study of the water wave problem in a two-dimensional domain in the presence of constant vorticity. The goal is to describe the effects of uniform shear flow on the modulation of weakly nonlinear quasi-monochromatic surface waves. Starting from the Hamiltonian formulation of this problem and using techniques of Hamiltonian transformation theory, we derive a Hamiltonian, high-order Nonlinear Schrödinger equation (often referred to as Dysthe equation) for the time evolution of the wave envelope. Consistent with previous studies, we observe that the uniform shear flow tends to enhance or weaken the modulational instability of Stokes waves depending on its direction and strength. This model is tested against direct numerical simulations of the full Euler equations and against a related Dysthe equation recently derived by Curtis, Carter and Kalisch (2018). This is a joint work with P. Guyenne and A. Kairzhan.

OLGA TRICHTCHENKO, Western University

THOMAS WOLF, Brock University

Exact solitary wave solutions for a coupled gKdV-NLS system

We study a coupled gKdV-NLS system $u_t + \alpha u^p u_x + \beta u_{xxx} = \gamma(|\psi|^2)_x,\ i\psi_t + \kappa\psi_{xx} = \sigma u\psi$ with nonlinearity power $p > 0$, which has been introduced in the literature to model energy transport in an anharmonic crystal material [1,2]. There is a strong interest in obtaining exact solutions describing frequency-modulated solitary waves $u = U(x - ct)$, $\psi = e^{i\omega t}\Psi(x - ct)$, with wave-speed c, and modulation frequency ω. Some solutions have been found for $p = 1$ (KdV) in [1], while for $p = 2$ (mKdV), no exact solutions were found [2]. Nothing has been done for $p \geq 3$.

We derived exact solutions for $p = 1, 2, 3, 4$, starting from the travelling wave ODE-system satisfied by U and Ψ. The method is new: (i) obtain first integrals by use of multi-reduction symmetry theory [3]; (ii) apply a hodograph transformation which leads to a triangular system; (iii) introduce an ansatz for polynomial solutions of the base ODE; (iv) characterize conditions under which solutions yield solitary waves; (v) solve an algebraic system for the unknown coefficients under those conditions.

The resulting solitary waves exhibit a wide range of features: bright/dark peaks; single/multi-peaked; zero/non-zero backgrounds.

References:

[1] L.A. Cisneros-Ake, J.F. Solano Pelaez, Bright and dark solitons in the unidirectional long wave limit for the energy transfer on anharmonic crystal lattices, Physica D 346 (2017), 20–27.

[2] L.A. Cisneros-Ake, H. Parra Prado, D.J. Lopez Villatoro, R. Carretero-Gonzalez, Multi-hump bright solitons in a Schrodinger–mKdV system, Physics Letters A 382 (2018), 837–845.

[3] S.C. Anco and M.L. Gandarias, Symmetry multi-reduction method for partial differential equations with conservation laws, Commun. Nonlin. Sci. Numer. Simul. 91 (2020), 105349.