THOMAS WOLF, Brock University

Exact solitary wave solutions for a coupled gKdV-NLS system

We study a coupled gKdV-NLS system $u_t + \alpha u^p u_x + \beta u_{xxx} = \gamma(|\psi|^2)_x,\ i\psi_t + \kappa\psi_{xx} = \sigma u\psi$ with nonlinearity power $p > 0$, which has been introduced in the literature to model energy transport in an anharmonic crystal material [1,2]. There is a strong interest in obtaining exact solutions describing frequency-modulated solitary waves $u = U(x - ct)$, $\psi = e^{i\omega t}\Psi(x - ct)$, with wave-speed c, and modulation frequency ω. Some solutions have been found for $p = 1$ (KdV) in [1], while for $p = 2$ (mKdV), no exact solutions were found [2]. Nothing has been done for $p \geq 3$.

We derived exact solutions for $p = 1, 2, 3, 4$, starting from the travelling wave ODE-system satisfied by U and Ψ. The method is new: (i) obtain first integrals by use of multi-reduction symmetry theory [3]; (ii) apply a hodograph transformation which leads to a triangular system; (iii) introduce an ansatz for polynomial solutions of the base ODE; (iv) characterize conditions under which solutions yield solitary waves; (v) solve an algebraic system for the unknown coefficients under those conditions.

The resulting solitary waves exhibit a wide range of features: bright/dark peaks; single/multi-peaked; zero/non-zero backgrounds.

References:

[1] L.A. Cisneros-Ake, J.F. Solano Pelaez, Bright and dark solitons in the unidirectional long wave limit for the energy transfer on anharmonic crystal lattices, Physica D 346 (2017), 20–27.

[2] L.A. Cisneros-Ake, H. Parra Prado, D.J. Lopez Villatoro, R. Carretero-Gonzalez, Multi-hump bright solitons in a Schrodinger–mKdV system, Physics Letters A 382 (2018), 837–845.

[3] S.C. Anco and M.L. Gandarias, Symmetry multi-reduction method for partial differential equations with conservation laws, Commun. Nonlin. Sci. Numer. Simul. 91 (2020), 105349.