BORYS KADETS, University of Georgia

Subspace configurations and low degree points on curves

The arithmetic irrationality $\mathrm{a.irr}_k X$ of a curve X over a number field k is the smallest integer d such that X has infinitely many points of degree d. Hyperelliptic curves $y^2 = f(x)$ of genus $g \geqslant 2$ have $\mathrm{a.irr}_k = 2$. Similarly, double covers of elliptic curves of positive rank have arithmetic irrationality 2; conversely, Harris and Silverman have shown that a curve with $\mathrm{a.irr}_k X = 2$ is geometrically hyperelliptic or bielliptic. Soon after Abramovich and Harris proved that a similar statement holds for curves with $\mathrm{a.irr}_k X = 3$. However, Debarre and Fahlaoui discovered that for all $d \geqslant 4$ there are families of curves with $\mathrm{a.irr}_k X = d$ which do not admit degree d or less maps to other curves. The existence of these Debarre-Fahlaoui curves makes it difficult to obtaining general results on curves with $\mathrm{a.irr}_k X = d$.

I will report on a recent joint work with Isabel Vogt (arXiv:2208.01067), in which we prove some results towards classifying curves of arithmetic irrationality d. We show that this classification problem can be reduced to a study of curves of low genus, and use this reduction to obtain a classification for $d \leqslant 5$. These results are obtained by studying a new discrete-geometric invariant — the subspace configuration — attached to curves of arithmetic irrationality d.