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Searching for Singularities in Navier-Stokes Flows Using Variational Optimization Methods

This investigation concerns a systematic computational search for potentially singular behavior in 3D Navier-Stokes flows. Enstrophy $\mathcal{E}(t)$ serves as a convenient indicator of the regularity of solutions — as long as this quantity remains finite, the solutions are guaranteed to be smooth and satisfy the equations in the classical sense. Another well-known conditional regularity result are the Ladyzhenskaya-Prodi-Serrin conditions asserting that the quantity $\mathcal{L}_{q,p} := \int_0^T \|\mathbf{u}(t)\|_{L^q(\Omega)}^p \, dt$, where $2/p + 3/q \leq 1$, $q > 3$, must remain bounded if the solution is smooth on the interval $[0, T]$. However, there are no finite a priori bounds available for these quantities and hence the regularity problem for the 3D Navier-Stokes system remains open. To quantify the maximum possible growth of $\mathcal{E}(T)$ and $\mathcal{L}_{q,p}$, we consider families of variational PDE optimization problems in which initial conditions are sought subject to certain constraints so that these quantities in the resulting Navier-Stokes flows are maximized. These problems are solved computationally using a large-scale adjoint-based gradient approach. By solving these problems for a broad range of parameter values we demonstrate that the maximum growth of $\mathcal{E}(T)$ and $\mathcal{L}_{q,p}$ appears finite. Thus, in the worst-case scenarios the two quantities remain bounded for all times and there is no evidence for singularity formation in finite time.

[Joint work with Dongfang Yun and Di Kang]