Algebraic Combinatorics and Representation Theory
Combinatoire algébrique et théorie des représentations

(Org: **Nantel Bergeron** and/et **Mike Zabrocki** (York))

FRANÇOIS BERGERON, UQAM

The Super ∇-Operator

I will describe a super version of the ∇ operator on the combinatorial Macdonald polynomials: $\widetilde{H}_\mu(\boldsymbol{x})$. For an extra set of variables $\boldsymbol{y}$, it is obtained by considering $\widetilde{H}_\mu(\boldsymbol{y})$ as an "eigenvalue" for $\widetilde{H}_\mu(\boldsymbol{x})$. After explaining why this new operator naturally contains and generalizes all instances of the operators involved in the Δ-conjecture, we will establish some of its properties. If time allows, we will discuss how to "lift" it to more parameters than just q and t, and associated implications regarding a bosonic-fermionic duality. This is joint work with J. Haglund, A. Iraci, and M. Romero.

KELVIN CHAN, York University

Recent progress on super harmonics

Super harmonics SH_n are anti-commuting variants of diagonal harmonics. The space SH_n is a symmetric group module whose dimension is conjectured to be counted by ordered set partitions. It is also conjectured by Zabrocki (arXiv:1902.08966) to yield a representation theoretic interpretation in superspace of the Delta Theorem at $t=0$. In this talk, we introduce a conjectured basis informed by a bijection on packed words and discuss some recent progress.

LUCAS GAGNON, York University

Unipotent symmetric functions

Symmetric functions are often thought of in relation to the representation theory of the symmetric groups, but they also have a representation theoretic connection to unipotent objects for the general linear groups over a finite field, $\mathrm{GL}_n(\mathbb{F}_q)$. In this talk I will describe how this connection can be used to realize two well known symmetric functions, the chromatic quasisymmetric function of an indifference graph and the unicellular LLT polynomial, via certain $\mathrm{GL}_n(\mathbb{F}_q)$ representations. The representations in question arise naturally from an investigation of the subgroup $\mathrm{UT}_n(\mathbb{F}_q)$ of unipotent upper triangular matrices, and this process suggests a more general method of constructing families of symmetric functions. As an added bonus, this construction also gives a new perspective on the relationship between chromatic quasisymmetric functions and unicellular LLT polynomials.

ANTHONY LAZZERONI, Hong Kong Baptist University

Powersum Bases in Quasisymmetric Functions and Quasisymmetric Functions in Non-commuting Variables

We introduce a new P basis for the Hopf algebra of quasisymmetric functions that refine the symmetric powersum basis. Unlike the quasisymmetric power sums of types 1 and 2, our basis is defined combinatorially: its expansion in quasisymmetric monomial functions is given by fillings of matrices. This basis has a shuffle product, a deconcatenate coproduct, and has a change of basis rule to the quasisymmetric fundamental basis by using tuples of ribbons. We lift our quasisymmetric powersum P basis to the Hopf algebra of quasisymmetric functions in non-commuting variables by introducing fillings with disjoint sets. This new basis has a shifted shuffle product and a standard deconcatenate coproduct, and certain basis elements agree with the fundamental basis of the Malvenuto-Reutenauer Hopf algebra of permutations.

BAPTISTE LOUF, Université du Québec à Montréal

GAYEE PARK, UQAM

Minimal skew semistandard Young tableaux and the Hillman–Grassl correspondence

Standard tableaux of skew shape are fundamental objects in enumerative and algebraic combinatorics and no product formula for the number is known. In 2014, Naruse gave a formula as a positive sum over excited diagrams of products of hook-lengths. In 2018, Morales, Pak, and Panova gave a q-analogue of Naruse's formula for semi-standard tableaux of skew shapes. They also showed, partly algebraically, that the Hillman-Grassl map restricted to skew shapes gave their q-analogue. We study the problem of making this argument completely bijective. For a skew shape, we define a new set of semi-standard Young tableaux, called the *minimal SSYT*, that are equinumerous with excited diagrams via a new description of the Hillman–Grassl bijection and have a version of excited moves. Lastly, we relate the minimal skew SSYT with the terms of the Okounkov-Olshanski formula for counting SYT of skew shape. This is joint work with Alejandro Morales and Greta Panova.

KEVIN PURBHOO, University of Waterloo

The MTV machine

Mukhin, Tarasov and Varchenko developed a "machine" for solving certain algebraic systems of equations, which arise in several places, including: Schubert calculus, algebraic curves, linear series, Wronskians of polynomials, mathematical physics. differential equations, and control theory. The machine is quite remarkable. Essentially, it transforms a hard system of algebraic equations into an easy system of differential equations. In this talk, I will attempt to explain how the machine works, and offer a new explanation for why it works.

FARHAD SOLTANI, York University

Quasisymmetric harmonics of the exterior algebra

We study the ring of quasisymmetric polynomials in n anticommuting (fermionic) variables. We show that the ideal generated by quasisymmetric polynomials is a commuting subalgebra and then we introduce a monomial basis which is indexed by ballot sequences for the quotient of the polynomial ring by the ideal generated by quasisymmetric polynomials.

ETIENNE TÉTREAULT, Université du Québec à Montréal

Plethystic decomposition of a power of homogeneous symmetric functions

The composition of (polynomial) representations of GL_n defines an operation, called plethysm, on associated characters. It is well-known that the decomposition of such a plethysm in irreducible characters is a hard problem, and we have no nice combinatorial description in general.

All this may be formulated in terms of symmetric functions, with Schur functions corresponding to irreducible characters. We consider the problem of decomposing, in the Schur basis, the plethysm $s_\mu[h_\lambda]$, where s_μ is a Schur function and h_λ a complete homogeneous symmetric function.

We approach this in the following way. Let m be an integer. We can write h_λ^m as a sum of plethysms $s_\mu[h_\lambda]$, one for each standard tableau of shape μ, for all partitions μ of m. Also, we know that the decomposition of h_λ^m is given by tableaux of content λ^m. Our conjecture is that the we can assign to thoses tableaux a type, which tells us in which plethysm the Schur function associated to this tableau appears. We show that to do so, we only need to consider h_n^m, and to construct what we call a Kronecker map, which involve a knowledge of the Kronecker coefficients.

In this talk, we quickly describe the problematic, and describe some exciting new advances toward the resolution of the conjecture. We also expose setbacks and limits which restrict us in our research.

NANCY WALLACE, York University

String decomposition of Parking functions

The search for the irreducible bicharacters for the space of diagonal harmonics, the search for a bijection that inverts the statistics area and bounce (or dinv) in the (q,t)-Caltalan formula of Garsia and Haiman or the (q,t)-Schröder formula of Haglund, and the more recent search for a basis for the diagonal harmonic alternants of Garsia and Zabrocki, all relate to the decomposition of these into strings that preserve the bidegree. In this talk we will give partial results on such a decomposition.

ALEX WILSON, Dartmouth College
A Diagram-Like Basis for the Multiset Partition Algebra

There's a classical connection between the representation theory of the symmetric group and the general linear group called Schur-Weyl Duality. Variations on this principle yield analogous connections between the symmetric group and other objects such as the partition algebra and more recently the multiset partition algebra. The partition algebra has a well-known basis indexed by graph-theoretic diagrams which allows the multiplication in the algebra to be understood visually as combinations of these diagrams. I will present an analogous basis for the multiset partition algebra and show how this basis can be used to describe generators and construct representations for the algebra.