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Plethystic decomposition of a power of homogeneous symmetric functions

The composition of (polynomial) representations of GL_n defines an operation, called plethysm, on associated characters. It is well-known that the decomposition of such a plethysm in irreducible characters is a hard problem, and we have no nice combinatorial description in general.

All this may be formulated in terms of symmetric functions, with Schur functions corresponding to irreducible characters. We consider the problem of decomposing, in the Schur basis, the plethysm $s_\mu[h_\lambda]$, where s_μ is a Schur function and h_λ a complete homogeneous symmetric function.

We approach this in the following way. Let m be an integer. We can write h_λ^m as a sum of plethysms $s_\mu[h_\lambda]$, one for each standard tableau of shape μ, for all partitions μ of m. Also, we know that the decomposition of h_λ^m is given by tableaux of content λ^m. Our conjecture is that the we can assign to thoses tableaux a type, which tells us in which plethysm the Schur function associated to this tableau appears. We show that to do so, we only need to consider h_n^m, and to construct what we call a Kronecker map, which involve a knowledge of the Kronecker coefficients.

In this talk, we quickly describe the problematic, and describe some exciting new advances toward the resolution of the conjecture. We also expose setbacks and limits which restrict us in our research.