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Multigrid proximal gradient method for convex optimization

We present a recent result on accelerating a 1st-order method for solving convex (possibly non-smooth) composite optimization problem of the form f(x) + g(x), where f is convex and differentiable and g is convex and possibly non-differentiable. We propose a multigrid (MG) based convergence acceleration method for the proximal gradient method. Coming from the domain of PDEs and scientific computing, the idea of multigrid assumes that the optimization problem has a hierarchical structure that can be exploited. By utilizing such hierarchy, acceleration can be achieved by a multi-level process.

We provide several theoretical results for the proposed method. We show a fixed-point property of the sequence generated by the method, and we provide a simple convergence analysis, based on a recent result on the Polyak-Lojasiewicz inequality, to show that the proposed method achieves a linear convergence rate on strongly convex problems.

Finally, we illustrate that the proposed MG-accelerated proximal gradient outperforms the proximal gradient method with Nesterov's acceleration, especially for large-sized problems in certain problem classes, such as a class of PDEs with a free boundary condition known as the elastic obstacle problem. If time permits, we will discuss briefly on the grid-independence convergence rate and also the on using MGProx for imaging application such as deblurrring.