KATARINA SACKA, McMaster University

Applications of Next-Iterate Operators to Discrete Planar Maps.

Two applications of next-iterate operators for discrete planar maps defined in the work by S.H. Streipert and G.S.K. Wolkowicz are explored. The time-delay equation

$$x_{n+1} = \frac{\alpha + x_{n-1}}{A + x_n}$$

for $n \in \mathbb{N}$, $\alpha \geq 0$, $A \in [0,1)$, $x_0 > 0$, and $x_1 > 0$ has a unique positive equilibrium which is a saddle point. Applying the change of variables, $y_n = x_{n-1}$, we write this equation as the planar system,

$$x_{n+1} = \frac{\alpha + y_n}{A + x_n}, \quad y_{n+1} = x_n.$$

We show that there exists a nontrivial positive solution which decreases monotonically to the equilibrium, proving Conjecture 5.4.6 from M. Kulenovic and G. Ladas. By using the augmented phase plane with nullclines and their associated root-curves, we can show the general behaviour of solutions in the plane. Using the tangent vector to the stable manifold at the equilibrium, we can show that solutions in a particular region defined by the nullclines and their associated root-curves, will decreases monotonically to the equilibrium along the tangent vector to the stable manifold. While Conjecture 5.4.6 has been previously proven, our proof provides a more elementary solution.

The second application of next-iterate operators regards the time delay equation,

$$x_{n+1} = \frac{\alpha + x_n + x_{n-1}}{A + x_n + x_{n-1}}$$

for $n \in \mathbb{N}$, $A > \alpha > 0$, $x_0 > 0$, and $x_1 > 0$. This equation has a unique positive equilibrium which is locally stable. Using the same change of variables as before, $y_n = x_{n-1}$, we write this equation as the planar system,

$$x_{n+1} = \frac{\alpha + x_n + y_n}{A + x_n + y_n}, \quad y_{n+1} = x_n.$$

By applying the augmented phase portrait, in addition to two new next-iterate operators defined in this work, we can expand this result to prove global stability.