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Deep neural Networks are effective at learning high-dimensional Banach-valued functions from limited data

Recently, there has been an increasing interest in applying Deep Learning (DL) to computational science and engineering, e.g., computer vision, genetics and computational uncertainty quantification (UQ). In particular, for UQ, high-dimensional problems are often posed in terms of parameterized partial differential equations (PDE) whose solutions take values in abstract spaces. Over the last five years, impressive results have been achieved on such problems using DL techniques, i.e., machine learning based on training Deep Neural Networks (DNN). However, little is known about the efficiency and reliability of DL from the perspectives of stability, robustness, accuracy, and sample complexity. This work focuses on approximating high-dimensional smooth functions taking values in a typically infinite-dimensional Banach space, where training data for such problems is often scarce and may be corrupted by errors. Moreover, obtaining samples is often expensive and involves a complicated black-box PDE solver and high problem dimensionality. Our results provide arguments for DNN approximation of such functions, with both known and unknown parametric dependence, that overcome the main challenge of the curse of dimensionality and account for all sources of error, i.e., sampling, optimization, approximation, and physical discretization. We assert the existence of a class of DNNs with dimension-independent architecture size and training procedures based on minimizing the regularized or unregularized ℓ_2-loss, which achieves near-optimal dimension-independent algebraic convergence rates. We provide numerical results illustrating the practical performance of DNNs on Hilbert-valued functions and preliminary numerical results on Banach-valued functions arising as solutions to parametric PDEs.