Mathematical Modeling and Analysis in Spatial Ecology and Epidemiology
Modélisation et analyse mathématiques en écologie et épidémiologie spatiales

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ELENA BRAVERMAN, University of Calgary

The influence of the choice of a diffusion strategy on the harvesting outcome for spatially heterogeneous populations

We describe a diffusion strategy as the tendency to have a distribution proportional to a certain positive prescribed function, once a diffusion coefficient grows infinitely. The talk is focused on the interplay of heterogeneity, variable diffusion strategies and populations' exploitation.

MICAH BRUSH, University of Alberta

Modelling long term mountain pine beetle dynamics with changing tree resilience

Over the last few decades, mountain pine beetle (MPB) have spread beyond their historical range into Alberta and threaten further spread North and East. This expansion has led MPB into novel species of pine as hosts, and their success in these species is not well understood. Climate change is also affecting pine resilience to MPB, particularly through increased drought. Accurate models predicting the long term dynamics of MPB in forests with changing tree resilience are therefore critical in assessing the risk of further expansion and informing management strategies.

In this talk, I will present a model that couples MPB population dynamics with forest growth that aims to understand how MPB dynamics will change on longer time scales and with different levels of host resilience. The model incorporates key aspects of MPB biology to realistically capture single outbreak behaviour, and has an age structured forest that regrows after MPB infestations. I will show that as forest resilience decreases, there is a fold bifurcation occurs and a stable fixed point appears with a non-zero MPB population. Simulations show that with initial conditions just above the Allee threshold, the number of beetles approaches this fixed point over a long time with transient outbreaks driven by the age structure of the forest. I will also show how adding a small number of lower vigor trees can lead to an additional stable fixed point with a small endemic population of beetles, and how with decreasing resilience can result in large outbreaks from this endemic population.

STEPHEN CANTRELL, University of Miami

Resource matching in spatial ecology and evolutionary advantage

A convergence of concepts from game theory (evolutionary stable strategy), ecological theory (the ideal free distribution), and mathematics (line sum-symmetry and its functional analytic generalizations) combine to explain how resource matching in spatially heterogeneous but temporally constant habitats can convey evolutionary advantage robustly across a range of modeling formats. The ideal free distribution (IFD) is an ecological construct that posits that when organisms have full knowledge of the landscape they inhabit (ideal) and are able to locate themselves as they wish (free), they will locate themselves to maximize reproductive fitness. The IFD can readily be translated into mathematical terms for models wherein the environment is spatially varying but temporally constant. In this talk we will discuss how this is done across a range of modeling formats and how it consequently leads to predictions of evolutionary advantage in such modeling formats. We then report on ongoing efforts to define the ideal free distribution mathematically in cases when the environment varies in both space and time, focusing on the case wherein habitats vary periodically in time.

YUMING CHEN, Wilfrid Laurier University

Global dynamics of an advective Lotka-Volterra competition-diffusion system

This talk is based on the Joint works with Professor De Tang. We consider a Lotka-Volterra competition-diffusion model in a one-dimensional advective environment. The two species are assumed to have the same population dynamics and advective rates but different diffusion rates. Moreover, the upstream end is supposed to be Neumann type boundary condition and the downstream end has a net loss of individuals measured by b. In the homogeneous case, if $0 < b < 1$, then the faster diffuser wins; if $b > 1$, then the slower diffuser wins (if it exists); and if $b = 1$, there is a compact global attractor consisting of a continuum of steady states. For the heterogeneous case, it is shown that the species with slower diffusion rate (if it exists) is always selected when $1 \leq b \leq \infty$.

FREDERIC HAMELIN, Institut Agro

Host Diversification May Split Epidemic Spread into Two Successive Fronts Advancing at Different Speeds

Host diversification methods such as within-field mixtures (or field mosaics, depending on the spatial scale considered) are promising methods for agroecological plant disease control. We explore disease spread in host mixtures (or field mosaics) composed of two host genotypes (susceptible and resistant). The pathogen population is composed of two genotypes (wild-type and resistance-breaking). We show that for intermediate fractions of resistant hosts, the spatial spread of the disease may be split into two successive fronts. The first front is led by the wild-type pathogen and the disease spreads faster, but at a lower prevalence, than in a resistant pure stand (or landscape). The second front is led by the resistance-breaking type, which spreads slower than in a pure resistant stand (or landscape). The wild-type and the resistance-breaking genotypes coexist behind the invasion fronts, resulting in the same prevalence as in a resistant pure stand. This study shows that host diversification methods may have a twofold effect on pathogen spread compared to a resistant pure stand (or landscape): on the one hand, they accelerate disease spread, and on the other hand they slow down the spread of the resistance-breaking genotype. This work contributes to a better understanding of the multiple effects underlying the performance of host diversification methods in agroecology. This is joint work with Y. Mammeri, Y. Aigu, S. E. Strelkov, and M. A. Lewis.

CHRISTOPHER HEGGERUD, UC Davis

Niche differentiation in the light spectrum promotes coexistence of phytoplankton species: a spatial modelling approach

The paradox of the plankton highlights the apparent contradiction between Gause's law of competitive exclusion and the observed diversity of phytoplankton. It is well known that phytoplankton dynamics depend heavily on two main resources: light and nutrients. Here we treat light as a continuum of resources rather than a single resource by considering the visible light spectrum and its attenuation through the water column. We propose a spatially explicit reaction-diffusion-advection model to explore under what circumstance coexistence is possible from mathematical and biological perspectives. Furthermore, we provide biological context as to when coexistence is expected based on the degree of niche differentiation within the light spectrum and overall turbidity of the water.

THOMAS HILLEN, University of Alberta

Pattern formation in non-local population models

Non-local advection is a key process in a range of biological systems, from cells within individuals to the movement of whole organisms. There has been increasing attention on pattern formation in non-local partial differential equations. The emergent patterns appear as local minimisers of a corresponding energy functional. Here we give approximate methods for determining the qualitative structure of local energy minimizers. These include a mixture of territory-like segregation patterns, full mixed cases, as well as narrow spike-type solutions. (joint work with V. Giunta, MA. Lewis, J. Potts)

THEODORE KOLOKOLNIKOV, Dalhousie

Modelling of disease spread through heterogeneous population

We present a simple model for the spread of an infection that incorporates spatial variability in population density. Starting from first-principle considerations, we explore how a novel partial differential equation with state-dependent diffusion can be obtained. This model exhibits higher infection rates in the areas of higher population density—a feature that we argue to be consistent with epidemiological observations. The model also exhibits an infection wave, the speed of which varies with population density. In addition, we demonstrate the possibility that an infection can 'jump' (i.e. tunnel) across areas of low population density towards areas of high population density. We briefly touch upon the data reported for coronavirus spread in the Canadian province of Nova Scotia as a case example with a number of qualitatively similar features as our model. Lastly, we propose a number of generalizations of the model towards future studies.

JUDE KONG, York University
Adaptive Changes in Sexual Behavior in Response to Monkeypox Can Control the Outbreak: Insights from an Epidemic Model

Monkeypox, a zoonotic disease caused by the monkeypox virus, is emerging as a potential sexually transmitted disease (STD). Starting from the end of April 2022, a monkeypox outbreak is ongoing. Mathematical modelling plays a crucial role in monitoring, controlling, and forecasting infectious disease outbreaks, including those generated by STDs. In this talk, I will present a compartmentalized epidemiological model that we designed to track the dynamics of Monkeypox and the results we obtained from analyzing the model. The model incorporates sexual behaviour dynamics and stratified the population into high- and low-risk groups. We explore and compare different intervention strategies targeting the high-risk population: i) a scenario of control strategies, implementing a policy geared towards the use of condoms and/or sexual abstinence (robust control strategy); ii) a scenario of control strategies with risk compensation behaviour change, assuming a compensation through conducting more sexual encounters for adopting protective behavioural strategies (risk compensation strategy); and, iii) a scenario of control strategies with behaviour change in response to the doubling rate (adaptive control strategy).

ADRIAN LAM, The Ohio State University
Invasion of open space by multiple competing species

I will discuss a question raised by Shigesada and Kawasaki concerning the stacked invasion fronts of two or more competing species on the real line when the initial values are null or exponentially decaying in a right half-line. In the case of compactly supported initial values, we prove that the first species spreads with the KPP speed of the single species, whereas the speed of the second species can be given by an exact formula depending on the speed of the first species. Generalization to three species, and the relation to the Fisher-KPP waves in shifting habitats will also be discussed. This is joint work with Leo Girardin (Lyons, France), Qian Liu (Shaoyang Univ.) and SHuang Liu (Beijing Inst. Tech., China).

MARK LEWIS, University of Victoria
Models and empirical evidence for the use of memory in animal movement patterns

Animal movement modelling provides unique insight about how animals perceive their landscape and how this perception influences their space use. This subject has recently been investigated by a variety of theoretical models from the perspective of pattern formation using coupled partial differential equation models. However, most of these models lack a solid empirical foundation. In this talk I focus on empirical evidence for the use of memory by animals while being tracked via radiotelemetry and how the data can incorporated into a step-selection function that can potentially connect back to partial differential equation models. I focus on patrolling behaviour in wolves (*Canis lupus*) in the foothills of the Rocky Mountains and on foraging behaviour in brown bears (*Ursus arctos*) in the Canadian Arctic.

BINGTUAN LI, University of Louisville
Effects of a barrier zone on invasion of a population with a strong Allee effect

We consider integro-difference and reaction-diffusion models to study the effects of a barrier zone on invasion of a population with a strong Allee effect. It is assumed that inside the barrier zone a certain proportion of the population is killed. We provide a formula for the critical width L* of barrier zone. We show that when a barrier zone is set properly at the front of a population, if the width of barrier zone is bigger than L* then the barrier zone can stop the population invasion, and if the width of barrier zone is less than L* then the population crosses the barrier zone and eventually occupies the entire space.

The results are proven by establishing the existence and attractivity of three types of equilibrium solutions. The mathematical proofs involve phase plane analysis and comparison.

MING MEI, Champlain College St-Lambert
Sharp traveling waves for degenerate viscosity Burgers equations with time-delay

In this talk, we are concerned with Burgers equations with degenerate viscosity and time-delay. The main issue is to investigate the structure of traveling waves. The waves are sharp caused by the degeneracy of viscosity, and oscillating caused by the large time-delay.

CHUNHUA OU, Memorial University of Newfoundland
Selection of the asymptotic spreading speed to the diffusive Lotka-Volterra competition model

In this talk, we will summarize our recent findings on the invasion speed (asymptotic spreading speed) to the the diffusive Lotka-Volterra competition model. This speed happens to be the minimal speed of traveling waves of the system, an important phenomenon in the study of mathematical biology, but the determinacy of the speed is challenging. By using both the parabolic and elliptic techniques in partial differential equations as well as a perturbation argument in a weighted functional space, we first derived a necessary and sufficient condition on the nonlinear selection which solves the conjecture raised by Roques et al. (J. Math. Biol., 2015). Furthermore, we established some easy-to apply criteria for linear and nonlinear selections in terms of lower (or upper) solutions with specific fast (slow) decay rate only, and we don't need to construct the couple of them simultaneously. This helped to easily obtain a number of explicit(analytic) results which gives estimates of the transition value in various situations. In particular, we established new results on the linear selection that doesn't require the system to be sub-linear on the direction of the positive eigenvector(Lewis, Li and Weinberger, 2002). Under certain conditions, we proved the Hosono's conjecture, but also pointed out failures of the conjecture in some cases. Our methods don't rely on the classical phase plane analysis and can be extended to work on any inhomogeneous system (including periodic systems and periodic habitats). This is a joint work of my team consisting of graduate students and visiting scientists.

SHIGUI RUAN, University of Miami
On the Dynamics of a Diffusive Foot-and-Mouth Disease Model with Nonlocal Infections

Foot-and-mouth disease (FMD) is an acute and highly contagious infectious disease of cloven-hoofed animals. In order to reveal the transmission dynamics and explore effective control measures of FMD, we formulate a diffusive FMD model with a fixed latent period and nonlocal infections. The threshold dynamics of the FMD model is determined by using the basic reproduction number $\mathcal{R}_0$: if $\mathcal{R}_0 < 1$ then the disease-free equilibrium E_0 is globally asymptotically stable; otherwise E_0 is unstable and there exists an endemic equilibrium E^*. Numerical simulations confirm the theoretical results and suggest that reducing the direct contact rate β_1 and the indirect contact rate β_2 is important in relieving FMD outbreaks. By carrying out some sensitivity analysis of $\mathcal{R}_0(> 1)$ and the equilibrium value of the infectious individuals I^* in terms of β_1 and β_2, it is found that the (β_1, β_2)-plane is divided into two regions by the intersection of two parameter-related surfaces, the sensitivity of $\mathcal{R}_0$ and I^* varies when β_1 and β_2 belong to different regions. When the values of both β_1 and β_2 are very large or very small, β_1 plays a more significant role on the transmission of FMD. These results indicate that stamping out the infected individuals and blocking the epidemic spots and areas are effective in preventing and controlling the spread of FMD.

YURIJ SALMANIW, University of Alberta
Modelling habitat loss with partial differential equations: the effects of habitat fragmentation on survival and abundance

It is well known that habitat loss is one of the major contributing factors to the decline of biodiversity worldwide. Partial differential equations offer one method to study the effects of habitat loss in a spatially explicit setting. Often, we identify three primary forms of habitat loss: degradation, destruction, and fragmentation. In this talk, I will briefly introduce two related competition diffusion systems subject to either habitat degradation or destruction. With these models as motivation, we shift our focus to the effects of habitat fragmentation through a careful study of the effect of habitat arrangement in two spatial dimensions. On one hand, we may consider the effect of fragmentation through survival of the population. This perspective allows one to define a quantitative measure of fragmentation. It is then possible to compare differing habitat arrangements of fixed volume. On the other hand, we may consider the effect of fragmentation through population abundance at the steady state. While this perspective does not lend itself to defining a measure of fragmentation, it does provide an interesting compliment to the first perspective. These mathematical insights in turn provide some interesting biological insights to the problem of fragmentation, and in fact highlight some key areas where confusion in the ecological literature may materialize.

ZHONGWEI SHEN, University of Alberta
Population dynamics under climate change

Reaction-diffusion equations in shifting environments are used to model the evolution of single species under climate change. Questions of theoretical importance are the effects of the shifting environment on the long-term dynamics of solutions. In this talk, I will report some relevant results about such equations without or with Allee effect.

JUNPING SHI, College of William & Mary
Evolution of dispersal in advective patchy environments

The classical Lotka-Volterra competition model predicts competition exclusion occurs when the competition is strong, and species can coexist when the competition is weak. In a spatially heterogeneous environment, the dispersal rates of species and the spatial heterogeneity could change or uphold the outcome of the competition. We show in a two-species Lotka-Volterra competition model in a patchy advective environment, where the species are subject to both directional drift and undirectional random dispersal between patches, under what conditions on the advection and random dispersal rates that a mutating species can or cannot invade the resident species. This is a joint work with Shanshan Chen, Zhisheng Shuai and Yixiang Wu.

OLGA VASILYEVA, Grenfell Campus, Memorial University of Newfoundland
Phase-plane analysis of steady states of a spruce budworm model with advection

The classical non-spatial Ludwig-Jones-Holling model and its reaction-diffusion version, the Ludwig-Aronson-Weinberger model, describe population dynamics of spruce budworm. Due to the complexity of the reaction term, under certain conditions, these models admit both endemic and outbreak steady state solutions. We explore the reaction-diffusion-advection version of the Ludwig-Aronson-Weinberger model, where advective term is interpreted as biased movement due to prevailing wind. Such a model can also describe other ecological settings where a logistically growing population is subject to diffusion, advection and predation by a generalist. We use phase-plane analysis to determine conditions for the existence of the outbreak solutions. In particular, we observe that increasing advection can prevent outbreaks while allowing persistence in the form of an endemic state. We obtain upper and lower bounds for the critical advection for outbreak steady state solutions. This is a joint work with Abby Anderson.

JAMES WATMOUGH, University of New Brunswick
Stage-structured dispersal in marine species with a pelagic larval stage.

Aquatic species often have radically different dispersal mechanisms at different life history stages. For example, a rapidly dispersing pelagic larval stage dispering in a current and a sessile or slow dispersing adult stage. We use a structured integro-difference equation model of the the spread of the green crab up the northwest coast of the Atlantic as a case study to examine the dependence of invasion dynamics on both adult and larval dispersal. Adding an additional dispersive stage increases invasion spread rates. However, it is unclear how the sensitivity of spread rate to underlying parameters might change with additional dispersive stages. Knowledge on the sensitivity of spread rate to demographic and dispersal parameters helps inform management strategies.

JIANHONG WU, York University

SHUWEN XUE, Northern Illinois University
Global existence, persistence and spreading speeds of a parabolic-parabolic chemotaxis model with logistic source

Chemotaxis models are used to describe the evolution of species in response to certain chemical substances in their living environments. In this talk, we will first introduce chemotaxis model. Then, we talk about the global existence and persistence of classical solutions under the condition that chemotaxis is small relative to the logistic damping. Next, under the same condition, we show that the spreading speed is the same as that of Fisher-KPP equation which implies that chemotaxis neither speeds up nor slows down the spatial spreading in the Fisher-KPP equation.

XINGFU ZOU, University of Western Ontario
Evolution of anti-predation response of prey in a general patchy environment

In this talk, I report some recent results on the evolution of anti-predation responses of a prey when perceiving the presence of its predator in a patch environment. To this end, we consider a ODE model on the patches in which two subspecies with distinct anti-predation response levels that affect the respective growths (cost) and predating rate (benefit) as well as their dispersion rates. We derive formulas for the invasion exponent and evolutionarily stable strategy. Our main techniques are from the theory of adaptive dynamics and a graph-theoretic approach based on the tree-cycle identity. In the scenario that the dispersion rate is increasing in fear level and the growth rate is decreasing in the fear level, our results indicate that the prey species with lower fear effect will invade in the heterogeneous environment. We also present some numerical simulations results to testify our theoretical findings, and discuss the effects of the monotonicity of mobility and fitness on evolutionarily stable strategy and convergence stable strategy. This is a joint work with Dan Huang.