Diophantine Arithmetic Geometry and Number Theory
Géométrie de l'arithmétique diophantienneet et Théorie des nombres
(Org: **Nathan Grieve** and/et **Patrick Ingram** (York))

JASON BELL,
Intersections of orbits of self-maps with subgroups in semiabelian varieties

Let G be a semiabelian variety defined over an algebraically closed field K, endowed with a rational self-map Φ. Let $\alpha \in G(K)$ and let $\Gamma \subseteq G(K)$ be a finitely generated subgroup. We show that the set $\{n \in \mathbb{N}\colon \Phi^n(\alpha) \in \Gamma\}$ is a union of finitely many arithmetic progressions along with a set of Banach density equal to 0. In addition, assuming Φ is regular, we prove that the set S must be finite.

MICHAEL GROECHENIG, University of Toronto
Arithmetic properties of rigid local systems

An irreducible local system is called rigid, if it cannot be deformed to a non-isomorphic local system. According to a conjecture by Simpson, rigid local systems on smooth projective varieties are expected to be of geometric origin, which leads to a swathe of surprising arithmetic and geometric properties for rigid local systems. In this talk I will explain how some of those properties can be established directly. This is joint work with Esnault.

KEPING HUANG, Michigan State University
Greatest Common Divisors on the Complement of Numerically Parallel Divisors

We prove inequalities involving greatest common divisors of functions at integral points with respect to numerically parallel divisors, generalizing a result of Wang and Yasufuku (after work of Bugeaud-Corvaja-Zannier, Corvaja-Zannier, and Levin). After applying a result of Vojta on integral points on subvarieties of semiabelian varieties, we use geometry and the theory of heights to reduce to the (known) case of $\mathbb{G}_m^n$. In addition to proving results in a broader context than previously considered, we also study the exceptional set in this setting, for both the counting function and the proximity function. This is a joint work with Aaron Levin.

BORYS KADETS, University of Georgia
Subspace configurations and low degree points on curves

The arithmetic irrationality $\mathrm{a.irr}_k X$ of a curve X over a number field k is the smallest integer d such that X has infinitely many points of degree d. Hyperelliptic curves $y^2 = f(x)$ of genus $g \geqslant 2$ have $\mathrm{a.irr}_k = 2$. Similarly, double covers of elliptic curves of positive rank have arithmetic irrationality 2; conversely, Harris and Silverman have shown that a curve with $\mathrm{a.irr}_k X = 2$ is geometrically hyperelliptic or bielliptic. Soon after Abramovich and Harris proved that a similar statement holds for curves with $\mathrm{a.irr}_k X = 3$. However, Debarre and Fahlaoui discovered that for all $d \geqslant 4$ there are families of curves with $\mathrm{a.irr}_k X = d$ which do not admit degree d or less maps to other curves. The existence of these Debarre-Fahlaoui curves makes it difficult to obtaining general results on curves with $\mathrm{a.irr}_k X = d$.

I will report on a recent joint work with Isabel Vogt (arXiv:2208.01067), in which we prove some results towards classifying curves of arithmetic irrationality d. We show that this classification problem can be reduced to a study of curves of low genus, and use this reduction to obtain a classification for $d \leqslant 5$. These results are obtained by studying a new discrete-geometric invariant — the subspace configuration — attached to curves of arithmetic irrationality d.

HYUNGSEOP KIM, University of Toronto
Thomason filtration via $T(1)$-local topological cyclic homology

Thomason's result tells us that algebraic K-theory of schemes on which p is invertible can be studied, after $T(1)$-localization (or morally etale sheafification), through p-adic etale cohomology. In this talk, I will explain how one can construct a filtration on $T(1)$-local TC of schemes through prismatic complexes, in a way compatible with Thomason's filtration.

DEBANJANA KUNDU, UBC Department of Mathematics
Studying Hilbert's 10th problem via explicit elliptic curves

N. García-Fritz and H. Pasten showed that Hilbert's 10^{th} problem is unsolvable in the ring of integers of number fields of the form $\mathbb{Q}(\sqrt[3]{p}, \sqrt{-q})$ for positive proportions of primes p and q. In joint work with A. Lei and F. Sprung, we improve their proportions and extend their results to the case of other number fields. We achieve this by replacing their Iwasawa theory arguments by a more direct method.

MATILDE LALIN, Université de Montréal
On the Northcott property for zeta functions over function fields and number fields

The Northcott property implies that a set of algebraic numbers with bounded height and bounded degree must be finite. Pazuki and Pengo introduced a variant of the Northcott property for number fields using special values of the Dedekind zeta function to measure the height. We consider this question for global function fields with constant field $\mathbb{F}_q$, evaluating the zeta function at any complex number. We also reconsider the question for Dedekind zeta functions with arbitrary evaluations. This is joint work with Xavier Généreux and Wanlin Li.

SUN KAI LEUNG, University of Montreal
Dirichlet law for factorization of integers, polynomials and permutations

Let $k \geq 2$ be an integer. We prove that factorization of integers into k parts follows the Dirichlet distribution Dir $\left(\frac{1}{k}, \ldots, \frac{1}{k}\right)$ by multidimensional contour integration, thereby generalizing the Deshouillers-Dress-Tenenbaum (DDT) arcsine law on divisors where $k = 2$. The same holds for the factorization of polynomials or permutations. Dirichlet distribution with arbitrary parameters can be modelled similarly. If time permits, we will also explore the evolution from the Dirichlet distribution to the multivariate normal distribution by restricting to smooth numbers.

WANLIN LI, Washington University in St. Louis
Ordinary Reductions of Abelian Varieties

Given an abelian variety A defined over a number field L, a conjecture attributed to Serre states that the density of primes of L at which A admits ordinary reduction is of positive density. This conjecture had been proved for elliptic curves (Serre, 1977), abelian surfaces (Katz 1982, Sawin 2016) and certain higher dimensional abelian varieties (Pink 1983, Fite 2021, etc). We will discuss some of the ideas behind these results and recent progress for abelian varieties with non-trivial endomorphisms, including the case of those with almost complex multiplication by an abelian CM field. This talk is based on joint work in progress with Victoria Cantoral-Farfan, Elena Mantovan, Rachel Pries, and Yunqing Tang.

JONATHAN LOVE, McGill University / CRM
On ℓ-torsion of superelliptic Jacobians over finite fields

For a prime $\ell \geq 3$, we study the ℓ-torsion subgroup of Jacobians J of curves $y^\ell = f(t)$ over a finite field $\mathbf{F}_q$. When $f(t)$ is a monic irreducible polynomial and q and $d := \deg(f)$ are both coprime to ℓ, we give an upper bound on the ℓ-rank of $J(\mathbf{F}_q)$ that depends only on q and d. Using tools from Galois cohomology, we prove that the ℓ-rank achieves this upper bound whenever $q^2 \equiv 1 \mod \ell$, and we find congruence conditions that can often be used to determine the ℓ-rank when the upper bound alone is not sufficient. This is joint work with Wanlin Li and Eric Stubley.

DAVID MCKINNON, University of Waterloo
Rational curves and rational points

Good approximations to rational points line up on rational curves. This, at least, is the conjecture, and it's been verified in a whole bunch of cases. In this talk, I'll discuss some cases where we know this, and even more cases where we could figure this out more if only we knew more about Vojta's Conjecture.

SIVA SANKAR NAIR, Université de Montréal
An Invariant Property of Mahler Measures

The Mahler measure of a polynomial $P(x_1, x_2, \ldots, x_n)$ is the average value of $\log|P|$ along the unit n-torus $\mathbb{T}^n$ (defined by $|x_i| = 1$ for all i). Interest in this quantity arose from the fact that Mahler measures of certain polynomials are quite remarkable and not just arbitrary real numbers. If P is univariate, this measure is given by Jensen's formula in terms of its roots, and in the multivariable case, it has been observed that it evaluates to special values of L-functions. Oftentimes, a numerical experiment leads to a conjecture equating the Mahler measures of certain polynomials to these special values. In this talk, we shall investigate an interesting invariant property that provides a method to extend identities involving Mahler measures and also resolve some conjectures along the way. This is joint work with Matilde Lalín.

MATT OLECHNOWICZ, University of Toronto
Dynamically improper hypersurfaces for endomorphisms of projective space

Most nonlinear endomorphisms of $\mathbb{P}^n$ have no nontrivial preperiodic subvarieties (that is, aside from preperiodic points and the whole space), which presents an obstacle to generalizing certain phenomena from $\mathbb{P}^1$. For instance, the statement that post-critically finite (PCF) maps are Zariski dense in the parameter space End_d^n (of degree-d endomorphisms of $\mathbb{P}^n$) is true when $n = 1$ but false when $n > 1$.

Motivated by these observations, we are led to consider an alternative generalization of the notion of preperiodicity, from points in $\mathbb{P}^1$ to hypersurfaces in $\mathbb{P}^n$, which we call dynamical improperness. In this talk, we will define what it means for a hypersurface to be dynamically improper and explain the connection to preperiodicity. We will show that every nonlinear endomorphism of $\mathbb{P}^n$ has infinitely many dynamically improper hypersurfaces. We will also show that maps with dynamically improper critical loci (which coincide with PCF maps when $n = 1$) are Zariski dense in the parameter space End_d^n for all $n > 1$ and $d > 2$.

SUBHAM ROY, Université de Montréal
Generalized Mahler measure of Laurent polynomials

The (logarithmic) Mahler measure of a non-zero rational polynomial P in n variables is defined as the mean of $\log|P|$ restricted to the standard n-torus ($\mathbb{T}^n = \{(x_1, \ldots, x_n) \in (\mathbb{C}^*)^n : |x_i| = 1, \forall\ 1 \leq i \leq n\}$). The Mahler measure has been related to special values of L-functions, and this has been explained in terms of regulators. In 2018, Lalín and Mittal considered the generalized Mahler measure (where the mean of $\log|P|$ is restricted to arbitrary n-torus) to obtain relations between certain polynomials mentioned in Boyd's paper. In this talk, we shall investigate the definition of the generalized Mahler measure for all Laurent polynomials in two variables when they do not vanish on the integration torus. We will then discuss few results we obtained involving the relation between the standard Mahler measure and the generalized Mahler measure of such polynomials.

RUIRAN SUN, CRM/McGill
Isotriviality of algebraic fiber spaces and the distribution of entire curves

It is conjectured that a quasi-projective manifold containing a non-degenerate entire curve is "special", and consequently any families of polarized manifolds over it should be isotrivial. In this talk we discuss a relative version of this "isotriviality conjecture". This is a joint work with Steven Lu and Kang Zuo.

SINA ZABANFAHM, University of Toronto
Cluster pictures for Hitchin fibers of rank two Higgs bundles

Let $\varphi\colon X \to Y$ be a degree two Galois cover of smooth curves over a local field F of odd characteristic. Assuming that Y has good reduction, we describe a semi-stability criterion for the curve X, using the data of the branch locus of the covering φ. In the case that X has semi-stable reduction, we describe the dual graph of the minimal regular model of X over F. We do this by adopting the notion of cluster picture defined for hyperelliptic curves for the case where Y is not necessarily a rational curve. Using these results, we describe the variation of the p-adic volume of Hitchin fibers over the semi-stable locus of the moduli space of rank 2 twisted Higgs bundles.

XIAO ZHONG, University of Waterloo
p-Adic interpolation of orbits under rational maps

Rivera-Letelier's characterization of possible analytic uniformizations of p-adic analytic maps has played an important role within arithmetic dynamics over the past fifteen years. The characterization is given by a trichotomy of indifferent, attracting and superattracting cases near a fixed point of a map.

In this talk, we present that if we are only interested in the orbit of a rational map on a point c of $\mathbb{P}^1$ over a characteristic zero global field, we could always p-adically interpolate the orbit in the sense similar to the indifferent case of the trichotomy. This is done by working with a finitely generated field extension of $\mathbb{Q}$ and choosing suitable primes for embedding into local fields. We also present an application to the dynamical Mordell-Lang conjecture.

This project is a joint work with Prof. Jason P. Bell (arxiv: 2202.01673).