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Approximate Solutions to the Superconducting Interface Model

The superconducting interface model is a semilinear hyperbolic system of PDEs introduced in 2016 by Kyle Thompson. It proposes a more tractable, yet closely related alternative to a 1984 model of Edward Witten for cosmic strings carrying a superconducting current. In the superconducting interface model, we consider the system

$$\begin{cases} \epsilon^2(\partial_t^2\varphi - \Delta_x\varphi) + \lambda_\varphi(\varphi^2-1)\varphi + \beta\varphi|\sigma|^2 = 0 \\ \epsilon^2(\partial_t^2\sigma - \Delta_x\sigma) + \lambda_\sigma(|\sigma|^2 - m_\sigma^2)\sigma + \beta\varphi^2\sigma = 0 \end{cases},$$

where $0 < \epsilon \ll 1$, $(\lambda_\varphi, \lambda_\sigma, m_\sigma, \beta) \in (0,\infty)^4$ are parameters, and

$$\varphi : [0,T]\times\mathbb{R}^n \to \mathbb{R} \text{ and } \sigma : [0,T]\times\mathbb{R}^n \to \mathbb{C},$$

for some $T > 0$ and $n \in \mathbb{N}$ with $n \geq 2$. We are interested in solutions (φ, σ) such that

$$\varphi \approx \begin{cases} +1 & \text{in } \mathcal{O}^+ \\ -1 & \text{in } \mathcal{O}^- \end{cases},$$

where $\mathcal{O}^+$ and $\mathcal{O}^-$ are disjoint open subsets of $[0,T]\times\mathbb{R}^n$ separated by an "interface" with thickness of order ϵ, and the current-carrying field σ decays exponentially away from the interface. The problem is of particular interest when σ showcases a clear interaction with the geometry of the interface.

The purpose of this talk is to present a methodology for finding solutions to the superconducting interface model by first constructing approximate solutions, and then linearizing the system of PDEs around these approximations. We will talk about some of the results regarding the construction of the approximate solutions and the laws of motion which represent the coupling between the current supported around the interface and the geometry of the interface.