Complex Geometry and Moduli Space
Géométrie complexe et espace de modules
(Org: **Michael Albanese** and/et **Ruxandra Moraru** (Waterloo))

TOM BAIRD, Memorial University
E-polynomials of character varieties associated to a real curve

Given a Riemann surface Σ denote by $M_n(\mathbb{F}) := Hom_\xi(\pi_1(\Sigma), GL_n(\mathbb{F}))/GL_n(\mathbb{F})$ the ξ-twisted character variety for $\xi \in \mathbb{F}$ a n-th root of unity. An anti-holomorphic involution τ on Σ induces an involution on $M_n(\mathbb{F})$ such that the fixed point variety $M_n^\tau(\mathbb{F})$ can be identified with the character variety of "real representations" for the orbifold fundamental group $\pi_1(\Sigma, \tau)$. When $\mathbb{F} = \mathbb{C}$, $M_n^\tau(\mathbb{C})$ is an ABA-brane: a half-dimensional complex subvariety of $M_n(\mathbb{C})$ which is sent to a Lagrangian submanifold of the moduli space of Higgs bundles under the non-abelian Hodge correspondence.

The E-polynomial of $M_n(\mathbb{C})$ was determined by Hausel and Rodriguez-Villegas by counting points in $M_n(\mathbb{F}_q)$ for finite fields $\mathbb{F}_q$. I will show how the same methods are used to calculate a generating function for the E-polynomial of $M_n^\tau(\mathbb{C})$ using the representation theory of $GL_n(\mathbb{F}_q)$.

ERIC BOULTER, University of Waterloo
Moduli Spaces of Sheaves on Kodaira Surfaces

Moduli spaces of stable sheaves on Kodaira surfaces give examples of compact holomorphic symplectic manifolds. The only known examples of non-Kähler holomorphic symplectic manifolds are Bogomolov-Guan manifolds or Douady spaces of points on Kodaira surfaces. In this talk we show that there exist compact moduli spaces in each even dimension, and that in the rank-2 case they are not Kähler but not deformation equivalent to Bogomolov-Guan manifolds. We also discuss some steps toward determining if these moduli spaces are deformation equivalent to Douady spaces of points on Kodaira surfaces.

BENOIT CHARBONNEAU, University of Waterloo
Symmetric instantons

During his MMATH study under my supervision, Spencer Whitehead developed a systematic framework to study instantons on R^4 that are invariant under groups of isometries. In this presentation, I will describe this framework and some results obtained using it.

XUEMIAO CHEN, University of Waterloo
Tangent cones of admissible Hermitian-Yang-Mills connections

Admissible Hermitian-Yang-Mills(HYM) connections are singular HYM connections with natural geometric bounds. In higher dimensional gauge theory, they naturally appear on the boundary of the moduli space of Hermitian-Yang-Mills connections over Kaehler manifolds. A fundamental problem was to study the uniqueness of the tangent cones of admissible HYM connections. I will explain joint work with Song Sun which confirms the uniqueness by showing that the tangent cones are algebraic invariants of the underlying reflexive sheaf.

CHANGHO HAN, University of Waterloo
Moduli of K3 surfaces with cyclic nonsymplectic automorphisms

K3 surfaces, as a 2-dimensional analog of elliptic curves, belong to an important class of varieties/complex-manifolds. Just as for the elliptic curves, K3 surfaces can be classified by using various invariants/viewpoints. In this talk, extending the idea from Alexeev and Engel for lattice-polarized K3 surfaces, I will explain how different viewpoints lead to different compactifications of the moduli of K3 surfaces with cyclic actions and then describe their birational relations. In particular, I will focus on the case of Kondo's sextic K3 surfaces and provide examples of boundary members of various compactifications. This talk is based on joint works in progress with Valery Alexeev, Anand Deopurkar, and Philip Engel.

CLAUDE LEBRUN, Stony Brook University
Twistors, Hyper-Kähler Manifolds, and Complex Moduli

A theorem of Kuranishi guarantees that the moduli space of complex structures on any smooth compact manifold is locally a finite-dimensional space. Globally, however, this finite-dimensionality can fail. Indeed, I will describe examples in which the moduli space contains a sequence of regions for which the local dimension tends to infinity. These examples, which naturally arise from the twistor theory of hyper-Kähler manifolds, also display other surprising behaviors. I will highlight several of these, and put the entire story in a context that contrasts high-dimensional complex manifolds with complex surfaces, and non-Kähler manifolds with complex algebraic varieties.

ALESSANDRO MALUSÀ, University of Toronto
Quantisation on hyper-Kähler spaces

Moduli spaces offer fertile ground for geometric quantisation. In that context, complex structures are commonly regarded as auxiliary data to be added to a symplectic form, the "true" classical structure. They also often come in families, and since they are extrinsic to quantisation one tries to remove them from the picture by constructing appropriate connections.

In the presence of a hyper-Kähler structure, however, there is no fixed underlying symplectic form: instead, there is a family of them, each coming with its own complex structure. In a joint work with J.E. Andersen and G. Rembado, we proposed a new approach to this problem, under sufficient symmetry assumptions, by introducing a holomorphic structure, rather than a connection, on the family of quantum Hilbert spaces, and tested it on a few interesting spaces. Furthermore, ongoing work with M. Mayrand has led to results in the case of Nahm moduli spaces, as well as insights on the problem of "quantisation commutes with reduction" for this new scheme.

In this presentation, I will give a panoramic of the new hyper-Kähler quantisation construction, and new results depending on time.

MAXENCE MAYRAND, Université de Sherbrooke
Twistor constructions of hyperkähler and hypercomplex structures near complex submanifolds

We discuss generalizations of the Feix-Kaledin theorem on the existence of hyperkähler structures on cotangent bundles of Kähler manifolds. Using twistor theory, we show that the problem of constructing a hyperkähler structure on a neighbourhood of a complex Lagrangian submanifold in a holomorphic symplectic manifold reduces to the existence of certain deformations of holomorphic symplectic structures. Similarly, hypercomplex structures near half-dimensional complex submanifolds can be constructed from certain deformations of complex structures. By combining these results with Hitchin's unobstructedness theorem on the deformation of holomorphic Poisson structures, we show that every holomorphic symplectic groupoid over a compact Kähler Poisson manifold has a hypercomplex structure on a neighbourhood of its identity section, and that there is a compatible hyperkähler metric if the Poisson manifold has complex dimension two.

BRENT PYM, McGill University
(Shifted) Poisson structures from noncommutative surfaces

I will describe a canonical, nonperturbative recipe for the deformation quantization of rational/ruled surfaces, obtained by twisting a natural semi-orthogonal decomposition of the derived category by a Morita automorphism of an anticanonical curve. The moduli spaces of "sheaves" on the resulting "noncommutative surfaces" have natural Poisson structures, generalizing the classical constructions of Bottacin and Mukai (in the commutative case), Nevins–Stafford (in the case of elliptic quantum planes) and more recent works of Rains (in the case of simple sheaves). The proof of the Jacobi identity for the Poisson bracket leverages and extends recent developments in the theory of shifted symplectic/Poisson structures in derived algebraic geometry, due to Brav–Dyckerhoff, (Calaque–)Pantev–Toën–Vaquié-Vezzosi, Melani–Safronov and Toën. This talk is based on joint work with Eric Rains.

ETHAN ROSS, University of Toronto
An Introduction to Stratified Vector Bundles

Stratified spaces are a class of singular spaces arising in contexts like algebraic geometry and equivariant topology. In this talk, I will be discussing a natural class of stratified spaces, namely stratified vector bundles. I will give three major classes of examples and two equivalent definitions.

CARLO SCARPA, CIRGET
Special representatives of complexified Kähler classes

Motivated by constructions appearing in mirror symmetry, we consider the problem of finding canonical representatives for a complexified Kähler class on a compact complex manifold. These are cohomology classes of the form $\beta + i\,\alpha$, for α a Kähler class and β an arbitrary real $(1, 1)$-class. As is often the case in complex geometry, one way to fix a representative of such a class is to impose an elliptic PDE. In this talk, I will explain why a natural choice of PDE is given by coupling the deformed Hermitian Yang-Mills equation and the constant scalar curvature equation. We will then see how to prove the existence of solutions in some special cases. Based on arXiv:2209.14157, joint work with Jacopo Stoppa.

XI SISI SHEN, Columbia University
The Continuity Equation on Hopf and Inoue Surfaces

We discuss the continuity equation of La Nave-Tian, extended to the Hermitian setting by Sherman-Weinkove, on Hopf and Inoue surfaces. We briefly outline the proof of a priori estimates for solutions in both cases, and Gromov-Hausdorff convergence of Inoue surfaces to a circle. This is joint work with Kevin Smith.

JEREMY USATINE, Brown University
Motivic integration for Artin stacks

A standard method for studying a singular variety is to resolve it by a smooth variety and to then relate invariants of the singular variety to invariants of the smooth one. Motivic integration provides powerful tools for obtaining such a relationship. Motivated by the McKay correspondence, I will describe a context in which interesting varieties admit natural resolutions of singularities by Artin stacks. This suggests a need for versatile tools in studying these "stacky" resolutions of singularities. I will discuss joint work with M. Satriano in which we use motivic integration to provide such tools, and I will also explain how our work leads to a notion of crepantness for stacky resolutions of singularities.