Analysis of PDEs
Analyse des EDP

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YEGANEH BAHOO, Toronto Metropolitan University
Visibility: Theory and Application

Computation Geometry is a recent branch in the field of Computer Science with roots established in ancient times, exploring geometric and numeric problems which arise from the constraints imposed by modern computing methods. Computational Geometry further serves as the foundation for a wealth of real-world applications reliant on efficient and elegant solutions.

In this talk, I will begin by presenting an overview of the field and my various contributions to it, with emphasis on visibility and pursuit-evasion problems These topics have significant research and industry interest due to their numerous applications, including wireless communications, robotics, computer graphics, and surveillance. Following this, I will discuss the interdisciplinary works which arise from close collaborations with engineering and machine learning groups, such as medical imaging, additive manufacturing, geometric deep learning.

The importance of the link between theory and application cannot be understated, as it is through the study of the theory that we can improve and expand the reach of applications; it is also through the present challenges faced in applications by which theoretical research can be informed. My ultimate goal is to extend the reach and relevance of Computational Geometry, and further its integration in new domains.

DER-CHEN CHANG, Georgetown University

MICHEL DELFOUR, Université de Montréal
Three-dimensional model of paclitaxel release from biodegradable polymer films

In order to achieve prescribed drug release kinetics in the wall and the lumen of blood vessels over long therapeutic periods, bi-phasic and possibly multi-phasic releases from blends of biodegradable polymers are currently envisioned. The modelling of drug release in the presence of degradation of the polymer matrix and surface erosion is quite complex. Yet, simple reliable mathematical models validated against experimental data are now available to classify neat polymers and to predict the release dynamics from polymer blends [Blanchet, Delfour, Garon, Quadratic models to fit experimental data of paclitaxel release kinetics from biodegradable polymers, SIAM J. on Applied Mathematics 71 (2011), 2269-2286]. We survey our two-parameter quadratic ODE model that has been validated against experimental data for the release of paclitaxel from a broad range of biodegradable polymers and our quadratic semi-permeable membrane PDE model that mimics the ODE model and readily extends to curved complex geometries of drug eluding stents [Garon, Delfour, Three-dimensional quadratic model of paclitaxel release from biodegradable polymer films, SIAM J. Appl. Math., 74 (5) (2014), 1354-1374]. This approach avoids resorting to time-dependent or nonlinear diffusion in the polymer. In the context of drug eluting stents, it is a practical and economical tool to theoretically and numerically simulate the 3D release of drug from the thin polymer film to the integrated wall and lumen of the blood vessel for evaluation and design [Delfour, Garon, Lamontagne, Three-Dimensional Drug Release in the Stent-Polymer-Wall-Lumen of a Blood Vessel, SIAM J. Appl. Math. 79 (2019), No. 5, 1850-1871].

OSCAR DOMINGUEZ-BONILLA, Universidad Complutense Madrid, CRM-Montreal
Functional&geometrical analysis of logarithmic Gagliardo-Lipschitz spaces

We present novel functional and geometrical aspects of logarithmic Gagliardo-Lipschitz function spaces, including capacities, perimeters and mean curvatures. This is joint work with Liguang Liu and Jie Xiao .

NGUYEN LAM, Memorial University

Sharp quantitative stability for the Uncertainty Principle

We present some sharp versions of the quantitative stability of the Heisenberg Uncertainty Principle and several stability results of the Caffarelli-Kohn-Nirenberg inequalities. The talk is based on a recent joint work with Cristian Cazacu, Joshua Flynn and Guozhen Lu.

MILIVOJE LUKIC, Rice University

An approach to universality using Weyl m-functions

This talk describes an approach to universality limits for orthogonal polynomials on the real line which is completely local and uses only the boundary behavior of the Weyl m-function at the point. We show that bulk universality of the Christoffel–Darboux kernel holds for any point where the imaginary part of the m-function has a positive finite nontangential limit. This approach is based on studying a matrix version of the Christoffel–Darboux kernel and the realization that bulk universality for this kernel at a point is equivalent to the fact that the corresponding m-function has normal limits at the same point. Our approach automatically applies to other self-adjoint systems with 2×2 transfer matrices such as continuum Schrodinger and Dirac operators. We also obtain analogous results for orthogonal polynomials on the unit circle. This is joint work with Benjamin Eichinger and Brian Simanek.

KODJO RAPHAEL MADOU, Université Laval

On the supercritical fractional diffusion equation with Hardy-type drift.

We study the heat kernel of the supercritical fractional diffusion equation with the drift in the critical Hölder space. We show that such a drift can have point irregularities strong enough to make the heat kernel vanish at a point for all $t > 0$.

The talk is based on joint work with D.Kinzebulatov and Yu.A. Semënov.

ROBERT MCCANN, University of Toronto

Asymptotics near extinction for nonlinear fast diffusion on a bounded domain

On a smooth bounded Euclidean domain, Sobolev-subcritical fast diffusion with vanishing boundary trace is known to lead to finite-time extinction, with a vanishing profile selected by the initial datum. In rescaled variables, we quantify the rate of convergence to this profile uniformly in relative error, showing the rate is either exponentially fast (with a rate constant predicted by the spectral gap), or algebraically slow (which is only possible in the presence of non-integrable zero modes). In the first case, the nonlinear dynamics are well-approximated by exponentially decaying eigenmodes up to at least twice the gap; this refines and confirms a 1980 conjecture of Berryman and Holland. We also improve on more recent results, by providing a new and simpler approach which is able to accommodate the presence of zero modes, such as those that occur when the vanishing profile fails to be isolated (and possibly belongs to a continuum of such profiles).

Based on work with Beomjun Choi (Postech) and Christian Seis (Muenster) [80] at http://www.math.toronto.edu/mccann/publications

MING MEI, Champlain College St-Lambert

Structural stability of subsonic steady-state for Euler-Poisson equations with sonic boundary

In this talk, I first review the subsonic/supersonic/transonic steady-states for Euler-Poisson equations for semiconductor device with sonic boundary, then I will present how these physical solutions are affected by the doping profile, and the structural stability of these steady-states with a small perturbation of the doping profile. The singularities for the structural stability come from the boundary and the transonic point at the sonic line. The weighted energy method is introduced to overcome the singularities.

MATHAV MURUGAN, The University of British Columbia
Harnack inequalities and conformal walk dimension

Harnack inequalities have proved to be a powerful tool in PDE (regularity estimates), geometry (geometric flows) and probability (heat kernel estimates). The notion of conformal walk dimension clarifies the relationship between elliptic and parabolic Harnack inequalities. I will explain its definition and its universal value. We will also discuss related results on the stability of Harnack inequalities. This talk is based on joint works with Martin Barlow, Zhen-Qing Chen and Naotaka Kajino.

JEREMY QUASTEL, University of Toronto
Integrable fluctuations in random growth

I will survey asymptotic fluctuations in the KPZ class and how they are connected to integrable equations.

SCOTT RODNEY, Cape Breton University
Bounded Solutions and Counterexamples

In this talk I will give a short overview of recent results with S.F. MacDonald concerning a priori boundedness of weak solutions to Dirichlet problems for operators

$$X_p u = -\mathrm{div}\left(\left|\sqrt{Q}\nabla u\right|^{p-2} Q\nabla u\right)$$

of p-Laplace type with a data function in an Orlicz class. Following this I will describe some relevant counterexamples.

ERIC SAWYER, McMaster University
Sums of squares of functions and matrices with application to hypoellipticity in the infinitely degenerate regime

This is joint work with Luda Korobenko. We extend the well-known theorem of Fefferman and Phong, that decomposes nonnegative C3,1 functions as finite sums of squares of C1,1 functions, to handle the case of C4,2delta functions. Additional assumptions are needed for this, and we give examples to demonstrate sharpness, in particular answering a question of Bony et al regarding elliptic such functions. These results are then extended to nonnegative matrices where they are applied to obtain new results on hypoellipticity of smooth infinitely degenerate operators. The techniques include extending a theorem of Mike Christ on sufficient conditions for smooth hypoellipticity to rough hypoellipticity, which is then applied back in the smooth case. The reason for the interest in decomposing nonnegative C4,2delta functions lies in the fact that the resulting sum is of squares of C2,delta functions, which have enough regularity to permit two differentiations in the case of second order rough operators.

ALEXEY SHEVYAKOV, University of Saskatchewan
Analytical Properties of Nonlinear Partial Differential Equations in Fluid Dynamics and Beyond

Analytical properties of partial differential equations (PDEs), in particular, models that arise in physics, engineering, and other applications, provide a fundamental counterpart to numerical solutions. Analytical methods for nonlinear PDEs have been under active development over the last hundred years; they include notions of S- and C-integrability and applications thereof, multiple other notions of integrability, Lagrangian and Hamiltonian structures, Painleve property, symmetries, conservation laws, reductions and exact solutions including solitons, and more.

In this talk we will look at some reductions of general fluid dynamics equations, including popular and less well known shallow water PDE models. Such models arise in a wide variety of settings within and beyond fluid surface waves. We will discuss some important analytical properties of such models with emphasis on those that are systematically computable. Examples of computation and applications of elements of analytical structure will be given for several PDE systems.

REIHANEH VAFADAR, Université Laval

On divergence-free (form-bounded type) drifts

We develop regularity theory for elliptic operator with drifts in a large class of divergence-free singular vector fields. A key ingredient of the proof is a new iteration procedure used within De Giorgi's and Moser's methods, playing the same role as e.g., the compensated compactness needed to handle BMO^{-1} drifts. This is joint work with Damir Kinzebulatov.

DEPING YE, Memorial University

The Minkowski type problems for unbounded convex hypersurfaces

A central object in convex geometry is the Minkowski problem which characterizes the surface area measure of convex bodies. This problem has been extended in various settings which all have close connections with partial differential equations (through the Monge-Ampere equations) and the optimal mass transport problem.

Recently, Schneider initiated the study of the Minkowski type problems for C-close sets, a family of (unbounded) closed convex sets contained in a cone. In this talk, I will talk about our recent progress on the Minkowski type problems for unbounded convex hypersurfaces. These Minkowski type problems generate new Monge-Ampere type equations. The solutions to these Minkowski type problems will also be presented.