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*Sums of squares of functions and matrices with application to hypoellipticity in the infinitely degenerate regime*

This is joint work with Luda Korobenko. We extend the well-known theorem of Fefferman and Phong, that decomposes nonnegative  $C^{3,1}$  functions as finite sums of squares of  $C^{1,1}$  functions, to handle the case of  $C^{4,2\delta}$  functions. Additional assumptions are needed for this, and we give examples to demonstrate sharpness, in particular answering a question of Bony et al regarding elliptic such functions. These results are then extended to nonnegative matrices where they are applied to obtain new results on hypoellipticity of smooth infinitely degenerate operators. The techniques include extending a theorem of Mike Christ on sufficient conditions for smooth hypoellipticity to rough hypoellipticity, which is then applied back in the smooth case. The reason for the interest in decomposing nonnegative  $C^{4,2\delta}$  functions lies in the fact that the resulting sum is of squares of  $C^{2,\delta}$  functions, which have enough regularity to permit two differentiations in the case of second order rough operators.