Advances in Finite Elements & Application to Solid and Fluid Mechanics
Progrès des éléments finis et application à la mécanique des solides et des fluides

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JAVIER ALMONACID, Simon Fraser University

Finite-element discretization of a 3D hyperelastic model of skeletal muscle

Recent studies on whole-muscle biomechanics have shown the importance of mass and inertial effects on muscle function. Because traditional models based on massless springs cannot capture these features, we must turn our attention to continuum-based three-dimensional models. In this talk, we will discuss the discretization process of a dynamical model that views skeletal muscle as a hyperelastic (nonlinear) deformable solid. From a mechanical perspective, this material is quasi-incompressible, transversely isotropic, and can be deformed by the action of active and passive forces. We will go over the different types of approximations (physiological and numerical) that must be considered to make the equations more tractable. The three-field formulation is discretized in space using a standard second-order finite element. In addition, we will discuss the Newton-Krylov strategy used to solve the set of nonlinear equations.

NICOLAS DOYON, Laval University

Finite element implementation of Poisson Nernst Planck equations in models of neural structures

Signaling in neural structures is determined by the movement of ions subjected to an electrical field which is best described by the Nernst Planck partial differential equations. The distribution of ionic concentrations in turns determines the electric field through the Poisson equation. The coupling of these equations gives rise to the Poisson Nernst-Planck (PNP) system of partial differential equations. To complete the picture, the opening of transmembrane channels describing the boundary conditions are often given by systems of ordinary differential equations involving the electrical field. In this talk, we present a model describing the evolution of ionic concentrations in a node of Ranvier using PNP equations together with Hodgkin-Huxley equations describing dynamics of transmembrane voltage-gated channels. Solving this model give rise to many numerical difficulties. For one, small imbalances in ionic concentrations can have a huge impact on the electrical field making it difficult to treat the problem as a fully coupled one. Second, the elongated geometries of structures such as axons or nodes of Ranvier makes difficult the construction of an efficient spatial mesh. Finally, the presence of a mostly impermeable membrane leads to solutions being non differentiable and exhibiting steep variations near the membrane cytosol interface. We will see how to tackle some these difficulties in particular by using automatic mesh adaptation. We will also discuss the relevance of related models and how they can be used in other contexts such as the description of cardiac cells and presynaptic vesicles.

KEEGAN KIRK, Rice University

Convergence analysis of a pressure-robust space-time HDG method for incompressible flows

Much of the recent progress in the numerical solution of incompressible flow problems has concentrated on pressure-robust finite element methods, a class of mimetic methods that preserve a fundamental invariance property of the incompressible Navier–Stokes equations. Two essential ingredients are required for pressure-robustness: exact enforcement of the incompressibility constraint, and H(div)-conformity of the finite element solution.

In this talk, I will introduce a space-time hybridized discontinuous Galerkin finite element method for the evolutionary incompressible Navier–Stokes equations. The numerical scheme has a number of desirable properties, including pointwise mass conservation, energy stability, and higher-order accuracy in both space and time. Through the introduction of a pressure facet variable, H(div)-conformity of the discrete velocity solution is enforced, ensuring the numerical scheme is pressure-robust.

Well-posedness of the resulting nonlinear algebraic system will be considered, and uniqueness of the discrete solution will be shown in two spatial dimensions under a small data assumption. A priori error estimates for smooth solutions will be presented, as well as convergence to weak solutions in the sense of Leray and Hopf using compactness results for discontinuous Galerkin methods.

LILIA KRIVODONOVA, University of Waterloo
Stabilization techniques for solution of hyperbolic conservation laws on unstructured nonconforming meshes

In order to resolve fine features of a numerical solution, run-time mesh refinement might be required. Commonly used refinement strategies aimed at preserving mesh quality result in nonconfirming meshes, i.e. meshes where a larger element might share an edge with several smaller elements. In this talk we will address solution stabilization techniques on such meshes using limiters. Limiting is a technique aimed at suppressing nonphysical oscillations in a numerical solution in the presence of shocks and steep solution gradients. Limiting on nonconforming meshes is difficult due to lack of structure in the mesh and because most limiting algorithms were developed for conforming meshes. The proposed limiter modifies solution coefficients (or moments) by reconstructing the slopes along a set of directions in which the moments decouple. The resulting solutions satisfy the local maximum principle (LMP) for scalar problems, i.e. are stable in the maximum norm. We show that our algorithm is efficient for solution of nonlinear hyperbolic systems such as Euler equations and scales well when implemented on GPUs.

JOSE PABLO LUCERO LORCA, University of Colorado at Boulder
Nonoverlapping Schwarz Preconditioners in linear and nonlinear settings applied to radiation transport problems.

We explore the application of multilevel, nonoverlapping domain decomposition to solve integro-differential problems of radiation transport colliding in media with the inclusion of a local thermodynamic equilibrium (LTE) nonlinearity. We discretize using discontinuous Galerkin finite elements, making the local problems small versions of the global problem.

By including a coarse space and minimizing the size of the local domains but ordering the application of the local solvers, we robustly achieve a constant amount of iterations for a fixed residual reduction in all regimes. We sequentially *sweep* local solves when collisions are low, and solve in parallel when they are high.

Our implementation takes advantage of the achievable parallelization while sweeping and complete parallelization while in the high-collision regime. With this preconditioner architecture, we apply the same philosophy for local non-linear solves, which are shown to be very effective for a local nonlinearity such as LTE and are promising for problems where the nonlinearity effect has a dominant direction.

CONOR MCCOID, Université Laval
Robust algorithm for the intersection of simplices

For some applications it is commonplace to use multiple grids in a single finite element solver. For example, in fluid-structure coupling a grid for the structure can be used separate from the grid for the fluid. This may occur as a 2D interface or, in the case of 3D mortar methods, the intersection of tetrahedral meshes. It is then essential to be able to project between simplicial meshes as robustly as possible. This talk presents an intersection algorithm designed to do just that between two simplicial grids in general dimension, making use of the principle of parsimony, with a focus on 2D and 3D grids.

SEBASTIAN DOMINGUEZ RIVERA, Siemens
Eigenvalues in linear elasticity: theory and approximation

In this talk we will discuss some eigenproblems regarding the Lamé operator for linear elasticity. Based on recent work, we consider different types of eigenvalue problems, including Steklov eigenvalues in elasticity, normal-tangential (where the normal component of the traction and the tangential component of the displacement are set to zero on the boundary) and tangential-normal (where the tangential component of the traction and the normal component of the displacement are set to zero on the boundary). We will cover theory, including some new types of Korn's inequality, and the approximation of these eigenpairs with the use of the finite element method.