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A proximal quasi-Newton trust-region method for nonsmooth regularized optimization

We develop a trust-region method for minimizing the sum of a smooth term f and a nonsmooth term h, both of which can be nonconvex. Each iteration of our method minimizes a possibly nonconvex model of $f + h$ in a trust region. The model coincides with $f + h$ in value and subdifferential at the center. We establish global convergence to a first-order stationary point when f satisfies a smoothness condition that holds, in particular, when it has Lipschitz-continuous gradient, and h is proper and lower semi-continuous. The model of h is required to be proper and lower-semi-continuous. Under these weak assumptions, we establish a worst-case $O(1/\epsilon^2)$ iteration complexity bound that matches the best known complexity bound of standard trust-region methods for smooth optimization. We detail a special instance in which we use a limited-memory quasi-Newton model of f and compute a step with the proximal gradient method, resulting in a practical proximal quasi-Newton method. We describe our Julia implementations and report numerical results on inverse problems from sparse optimization and signal processing. Our trust-region algorithm exhibits promising performance and compares favorably with linesearch proximal quasi-Newton methods based on convex models.