Non-local PDEs in Biological Modelling
EDPs non-locales en modélisation biologique
(Org: **Dr. Thomas Hillen** (Alberta))

ANDREAS BUTTENSCHOEN, University of British Columbia
Non-Local Cell Adhesion Models: Symmetries and Bifurcations in 1-D

In both normal tissue and disease states, cells interact with one another, and other tissue components using adhesion proteins. These interactions are fundamental in determining tissue fates, and the outcomes of normal development, and cancer metastasis. Traditionally continuum models (PDEs) of tissues are based on purely local interactions. However, these models ignore important nonlocal effects in tissues, such as long-ranged adhesion forces between cells.

In this talk, I focus on the nonlocal "Armstrong adhesion model" (2006) for adhering tissue (an example of an aggregation equation). Since its introduction, this approach has proven popular in applications to embyonic development and cancer modeling. Combining global bifurcation results pioneered by Rabinowitz, equivariant bifurcation theory, and the mathematical properties of the non-local term, we prove a global bifurcation result for the non-trivial solution branches of the scalar Armstrong adhesion model. I will demonstrate how we used the equation's symmetries to classify the solution branches by the nodal properties of the solution's derivative.

Joint work with Thomas Hillen (University of Alberta).

RALUCA EFTIMIE, University of Franche-Comté
Pattern formation in nonlocal mathematical models for cancer invasion

Cancer invasion is a complex process involving interactions between cancer cells, between cells and extracellular matrix, and between cancer cells and various immune cells inside the tumour microenvironment. These interactions are facilitated by different molecules (e.g., cytokines, chemokines), which can be produced by the cancer cells themselves, or by other cells in the microenvironment. Here we introduce a class of nonlocal mathematical models for cancer proliferation and invasion (models described by hyperbolic and/or parabolic PDEs), and investigate analytically and numerically the spatial and spatio-temporal patterns exhibited by these models.

RAZVAN FETECAU, Simon Fraser University
Collective behaviour with intrinsic interactions on Riemannian manifolds

We consider a model for collective behaviour with intrinsic interactions on Riemannian manifolds. Such models can have applications in biology or robotics, where biological agents/robots are restricted by environment or mobility constraints to remain on a certain manifold. We provide a framework for constructing interaction potentials which lead to equilibria that are constant on their supports. We consider such potentials for two specific cases (the two-dimensional sphere and the two-dimensional hyperbolic space) and investigate analytically and numerically the long-time behaviour and equilibrium solutions.

JIMMY GARNIER, CNRS - University Savoie Mont-Blanc
Evolutionary dynamics of populations : nonlocal PDEs approaches

In this talk I will present some results about evolutionary dynamics of populations using nonlocal PDEs. More precisely, I will focus on the evolution of sexual or asexual population facing environmental change. Starting with a Individual Based Model, we obtain an analytical description of this microscopic model using nonlocal partial differential equations. In a special regime of small variance, we are able to approximate analytically the behavior of the microscopic model and we deduce qualitative as well as quantitative effects of the environmental change on the evolutionary dynamics of the population.

VALERIA GIUNTA, University of Sheffield

Calculating the stable steady states of multispecies non-local advection-diffusion models using energy functionals

In many biological systems, it is essential for individuals to gain information about their local environment, not only to sense environmental features, such as food, but also to detect other individuals in a local spatial neighborhood. Interestingly, this feature is not only restricted to higher level species, such as animals, but is also found in cells, which interact non-locally by extending long thin protrusions, probing the environment.

The process of gaining information about presence of other species in the environment is intrinsically non-local and mathematically the non-local sensing of neighboring individuals leads to non-local advection terms in continuum models.

In this talk, I will focus on a class of nonlocal advection-diffusion equations modeling population movements generated by inter- and intra-species interactions. After a brief discussion of the well-posedness of this problem, I will show that the model supports a great variety of spatio-temporal patterns, including stationary aggregations, segregations, oscillatory patterns, and irregular spatio-temporal solutions. However, if populations respond to each other in a symmetric fashion, linear stability analysis shows that the only patterns that emerge from small perturbations of the stable steady state are stationary. In this case, the system admits an energy functional that is decreasing and bounded below, suggesting that patterns remain stationary for all time. I will show how to use this functional to gain insight into the analytic structure of the stable steady state solutions. This procedure reveals a range of possible stationary patterns, including various multi-stable situations, which we validate via comparison with numerical simulations.

RISHABH GVALANI, Max Planck Institut

The McKean–Vlasov equation on the torus: Stationary solutions, phase transitions, and mountain passes.

We study the McKean–Vlasov equation on the torus which is obtained as the mean field limit of a system of interacting diffusion processes enclosed in a periodic box. We focus our attention on the stationary problem - under certain assumptions on the interaction potential, we show that the system exhibits multiple equilibria which arise from the uniform state through continuous bifurcations. We then attempt to classify continuous and discontinuous phase transitions for this system and show that at the point of discontinuous transition the free energy possesses a mountain pass point. Finally, we comment on further work generalising these results to equations with porous medium-type diffusion. Joint work with José A. Carrillo, Greg Pavliotis, and André Schlichting.

FLORIAN PATOUT, BioSP, INRAE

Ancestral lineages in mutation selection equilibria with moving optimum

Many populations can somehow adapt to rapid environmental changes. To understand this fast evolution, we investigate the genealogy of individuals inside those populations. More precisely, we use a deterministic model to describe the phenotypic density of a population under selection when the fitness optimum moves at constant speed. We study the inside dynamics of this population using the neutral fractions approach. We then define a Markov process characterizing the distribution of ancestral phenotypic lineages inside the equilibrium. This construction yields qualitative as well as quantitative properties on the phenotype of typical ancestors. In particular, we show that in asexual populations typical ancestors of present individuals carried traits much closer to the fitness optimum than most individuals alive at the same time. We also investigate more deeply the asymptotic regime of small mutation effects. In this regime, we obtain an explicit formula for the typical ancestral lineage using the description of the solutions of Hamilton Jacobi equation as a minimizer of an optimization problem. In addition, we compare our deterministic results on lineages with the lineages of stochastic models.

ANGELA STEVENS, University of Münster

Attractive-Repulsive Interactions

Connections between chemotaxis-like equations and non-local equations are discussed and the support of the respective solutions are analyzed. One interesting phenomenon for e.g. two interacting species is their segregation (Joint works with M. Burger,

M. Di Francesco, S. Fagioli, with G. Kaib, K. Kang and with M. Winkler)

VITALI VOUGALTER, University of Toronto
Solvability of some integro-differential equations with drift

We prove the existence in the sense of sequences of solutions for some integro-differential type equations involving the drift term in the appropriate H^2 spaces using the fixed point technique when the elliptic problems contain second order differential operators with and without Fredholm property. It is shown that, under the reasonable technical conditions, the convergence in L^1 of the integral kernels yields the existence and convergence in H^2 of solutions.