Harmonic analysis and fractal geometry
Analyse harmonique et géométrie fractale
(Org: **Malabika Praminak** (UBC) and/et **Alexia Yavicoli**)

BENJAMIN BRUCE, University of British Columbia
Scale-invariant restriction estimates for some hyperbolic surfaces

I will discuss scale-invariant Fourier restriction estimates for two hyperbolic surfaces. One is the "hyperbolic cone" in $\mathbb{R}^4$, a conic surface whose cross sections are hyperbolic paraboloids. The other is the one-sheet hyperboloid in $\mathbb{R}^3$. The geometric structure of these surfaces, especially the presence of many lines, plays a large role in the proof of the estimates. Partially joint work with Diogo Oliveira e Silva and Betsy Stovall.

GALIA DAFNI, Concordia University
Inhomogeneous cancellation conditions and Calderón–Zygmund type operators on local Hardy spaces

In joint work with Chun Ho Lau (Concordia), Tiago Picon (Universidade São Paulo), and Claudio Vasconcelos (Universidade Federal de São Carlos), we present a new approach to atoms and molecules on Goldberg's local Hardy spaces $h^p(\mathbb{R}^n)$, $0 < p \leq 1$, assuming appropriate cancellation conditions. As applications, we prove a version of Hardy's inequality and improved continuity results for inhomogeneous Calderón–Zygmund operators on these spaces.

FRANCESCO DI PLINIO, Washington University in St. Louis
Maximal Subspace Averages

We study maximal operators associated to singular averages along finite subsets of the Grassmannian of d-dimensional subspaces of the n-dimensional Euclidean space. The well studied $d = 1$ case corresponds to the usual directional maximal function. We provide a systematic study of all cases $1 \leq d < n$ and prove essentially sharp L^2 bounds for the maximal subspace averaging operator in terms of the cardinality of the finite subset without any assumption on the structure. In the codimension 1 case, that is $n = d + 1$, we prove the precise critical weak $(2, 2)$-bound. Our estimates rely on Fourier analytic almost orthogonality principles, combined with polynomial partitioning, but we also use spatial analysis based on the precise calculation of intersections of d-dimensional plates. Joint work with Ioannis Parissis, University of Basque Country and IkerBasque.

JENNIFER DUNCAN, University of Birmingham
Some Global Results on Nonlinear Brascamp-Lieb Inequalities

The Brascamp-Lieb inequalities are a powerful generalisation of many classical multilinear inequalities, such as Hölder's inequality, Young's convolution inequality, and the Loomis-Whitney inequality, as well as a fundamental manifestation of a notion of transversality that commonly arises in multilinear harmonic analysis. In this talk, I will introduce the linear and nonlinear Brascamp-Lieb inequalities, explain their relation to topics such as multilinear Kakeya and Fourier restriction, and discuss some of the results I have proved over the course of my PhD, which aim to address the question of nonlinear Brascamp-Lieb inequalities over unbounded domains.

SURESH ESWARATHASAN, Dalhousie University
Fractal uncertainty principle for discrete Cantor sets for random alphabets

The fractal uncertainty principle (FUP) introduced by Dyatlov-Zahl '16 has seen some powerful applications in the last few years and become a hot topic in harmonic analysis. In this talk, we study the FUP for discrete Cantor sets from a probabilistic perspective. We show that randomizing our alphabets gives a quantifiable improvement over the current "zero" and "pressure" bounds. In turn, this provides the best possible exponent when the Cantor sets enjoy either the strongest Fourier decay or additive energy assumptions. This is joint work with Xiaolong Han (Cal. State Northridge)

JONATHAN FRASER, University of St Andrews
Assouad dimension of distance sets

The 'distance set problem' concerns understanding the relationship between the size (e.g. dimension) of a set $F \subseteq \mathbb{R}^d$ and the associated 'distance set' $\{|x-y| : x, y \in F\}$. I will discuss recent progress on the Assouad dimension version of this problem.

MICHAEL HOCHMAN, The Hebrew University
Fractal methods in equidistribution

I will discuss some methods for establishing equidistribution results for a.e. point of a measure based on the fractal structure of the measure.

MARINA ILIOPOULOU, University of Kent
Sharp L^p estimates for oscillatory integral operators of arbitrary signature

The restriction problem in harmonic analysis asks for L^p bounds on the Fourier transform of functions defined on curved surfaces. In this talk, we will present improved restriction estimates for hyperbolic paraboloids, that depend on the signature of the paraboloids. These estimates still hold, and are sharp, in the variable coefficient regime. This is joint work with Jonathan Hickman.

VJEKOSLAV KOVAČ, University of Zagreb
Density theorems for anisotropic point configurations

Several mathematical areas search for patterns in large, but otherwise arbitrary structures. Euclidean density theorems seek for dilated, rotated, and translated copies of a fixed finite point configuration within a "large" subset of the Euclidean space of appropriate dimension, where "largeness" is then defined as having a strictly positive (appropriately defined) density. Of particular interest are results that identify the configuration dilated by all sufficiently large scales; a line of research proving such results extends back to the 1980s and, so far, all results of this type discussed linear isotropic dilates of a fixed point configuration. We will report on the beginnings of the study of analogous density theorems for families of point configurations generated by anisotropic dilations, i.e., families with power-type dependence on a single parameter interpreted as their size. At the same time, we will single out anisotropic multilinear singular integral operators associated with these combinatorial problems, as they are interesting on their own. We will also mention related author's joint work with P. Durcik, K. Falconer, L. Rimanić, and A. Yavicoli.

DOMINIQUE MALDAGUE, MIT
Small cap decoupling for the cone in $\mathbb{R}^3$

Decoupling involves taking a function with complicated Fourier support and measuring its size in terms of projections onto easier-to-understand pieces of the Fourier support. A simple example is periodic solutions to the Schrodinder equation in one spatial and one time dimension, which have Fourier series expansions with frequency points on the parabola (n, n^2). The exponential sum (Fourier series) itself is difficult to understand, but each summand is very simple. I will explain the basic statement of decoupling and the tools that go into the high/low frequency approach to its proof, focusing on upcoming work in collaboration with Larry Guth concerning the cone in $\mathbb{R}^3$. Our work further sharpens the refined L^4 square function estimate for the truncated cone $C^2 = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2, \quad 1/2 \leq |z| \leq 2\}$ from the local smoothing paper of L. Guth, H. Wang, and R. Zhang. A corollary is sharp (ℓ^p, L^p) small cap decoupling estimates for the cone C^2, for the sharp range of exponents p. The base case of the "induction-on-scales" argument is the corresponding sharpened, refined L^4 square function inequality for the parabola, which leads to a new proof of canonical (Bourgain-Demeter) and small cap (Demeter-Guth-Wang) decoupling for the parabola.

EYVINDUR PALSSON, Virginia Tech
Triangles and the triangle averaging operator

Point configuration questions can give rise to geometric averaging operators. A natural example is the spherical averaging operator, which can encode the simple point configuration that is given by distance. In this talk I will introduce a multilinear analogue, the triangle averaging operator and discuss some recent work on its mapping properties. If time permits I will also discuss a recent Mattila-Sjolin type theorem for triangles.

LENKA SLAVIKOVA, Charles University
Local bounds for singular Brascamp-Lieb forms with cubical structure

In this talk we discuss the boundedness properties of certain multilinear forms that involve a Calderón-Zygmund kernel and possess a cubical structure. Special instances of these forms have found applications in enumerative combinatorics and ergodic theory. Passing through local and sparse bounds, we prove a range of L^p bounds for these forms, extending thus an earlier result by Durcik and Thiele which only allowed for one particular tuple of exponents. New in this context is the use of a modified strong maximal function. This is a joint work with P. Durcik and C. Thiele.

VILLE SUOMALA, University of Oulu
Fractal percolation, points, and lines

I will discuss the following phase transitions for the dyadic fractal percolation in the plane: If the retention probability p is at most $2^{(-k-2)/k}$ then each line contains at most $k-1$ points of the fractal percolation. If $p > 2^{(-k-2)/k}$, then some line contains k points of fractal percolation.

KRYSTAL TAYLOR, The Ohio State Math Department
Newhouse Thickness and Falconer-type problems

Th Falconer distance conjecture states that if E is a subset of R^d of Hausdorff dimension greater then $d/2$, then the set of distances $\Delta(E) = \{|x-y| : x, y \in E\}$ has positive measure. Related questions include finding conditions on E which guarantee that (1) $\Delta(E)$ has non-empty interior or; (2) E contains various finite point configurations. We consider a variant of these problems in which the Hausdorff dimension is replaced by an alternate notion of structure, mainly that of Newhouse thickness, and the single distance $|x-y|$ is replaced by a tuple of distances described by a given graph over E.

HONG WANG, UCLA
Distance sets spanned by sets of dimension d/2

If E is a set of Hausdorff dimension $d/2$ on $\mathbb{R}^2$, how large is its distance set $\Delta(E) := \{|x-y|, x, y \in E\}$. It turns out that the problem is quite different from the one about sets of dimension $> d/2$. We will explain the difficulties and our results.

This is joint work with Pablo Shmerkin.

JOSHUA ZAHL, University of British Columbia
The dimension of exceptional parameters for nonlinear projections, and the discretized Elekes-Rónyai theorem.

I will discuss exceptional parameters for nonlinear analogues of the (linear) projection function $\pi_\theta(x,y) = x\cos\theta + y\sin\theta$. A direction θ is called exceptional for a set A if the dimension of the projection $\pi_\theta(A)$ is smaller than the dimension of the projection in a "typical" direction. Results of Kaufman and Bourgain quantify the size of the set of exceptional directions. There are several results concerning nonlinear generalizations of the projection function π_θ. The theme of these results is that under suitable constraints, nonlinear projections are as well-behaved as linear projections, with respect to the size of the set of exceptional "directions." I will discuss a new and unexpected phenomena, which says that in general, nonlinear projections are much better behaved than linear ones. This is joint work with Orit Raz.