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Small cap decoupling for the cone in $\mathbb{R}^3$

Decoupling involves taking a function with complicated Fourier support and measuring its size in terms of projections onto easier-to-understand pieces of the Fourier support. A simple example is periodic solutions to the Schrodinder equation in one spatial and one time dimension, which have Fourier series expansions with frequency points on the parabola (n, n^2). The exponential sum (Fourier series) itself is difficult to understand, but each summand is very simple. I will explain the basic statement of decoupling and the tools that go into the high/low frequency approach to its proof, focusing on upcoming work in collaboration with Larry Guth concerning the cone in $\mathbb{R}^3$. Our work further sharpens the refined L^4 square function estimate for the truncated cone $C^2 = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2, \quad 1/2 \leq |z| \leq 2\}$ from the local smoothing paper of L. Guth, H. Wang, and R. Zhang. A corollary is sharp (ℓ^p, L^p) small cap decoupling estimates for the cone C^2, for the sharp range of exponents p. The base case of the "induction-on-scales" argument is the corresponding sharpened, refined L^4 square function inequality for the parabola, which leads to a new proof of canonical (Bourgain-Demeter) and small cap (Demeter-Guth-Wang) decoupling for the parabola.