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The L_p Brunn-Minkowski theory for C-coconvex sets

Recently, Schneider built up the Brunn-Minkowski theory for C-coconvex sets. Unlike convex bodies which are compact convex sets with nonempty interiors in $\mathbb{R}^n$, C-coconvex sets are non-compact convex sets contained in some given pointed closed convex cone in $\mathbb{R}^n$ (a cone with vertex at the origin and having nonempty interior). Schneider obtained many fundamental results on C-coconvex sets, such as the existence of the solutions to the Minkowski problem and the log-Minkowski problem for C-conconvex sets.

In this talk, I will discuss how an L_p Brunn-Minkowski theory can be developed. In particular, I will explain the L_p addition of C-coconvex sets and the related variational formula in terms of volume. I will present the L_p Brunn-Minkowski and L_p Minkowski inequalities for C-coconvex sets. I will also discuss the L_p Minkowski problem for C-coconvex sets (including the existence and uniqueness of its solutions). The case when $p = 0$ is of particular interest, because the log-Brunn-Minkowski and log-Minkowski inequalities can be proved. These results can be applied to the log-Minkowski problem for C-coconvex sets to obtain the uniqueness of solutions to this problem–hence solved an open problem raised by Schneider. Note that the log-Brunn-Minkowski and log-Minkowski inequalities (hence the uniqueness of solutions to the log-Minkowski problem for convex bodies) are still quite open.