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On Hadwiger's Conjecture

In 1943, Hadwiger conjectured that every graph with no K_t minor is $(t-1)$-colorable for every $t \geq 1$. Hadwiger's Conjecture is a vast generalization of the Four Color Theorem and one of the most important open problems in graph theory. Only the cases when t is at most 6 are known. In the 1980s, Kostochka and Thomason independently proved that every graph with no K_t minor has average degree $O(t(\log t)^{0.5})$ and hence is $O(t(\log t)^{0.5})$-colorable. In a recent breakthrough, Norin, Song, and I proved that every graph with no K_t minor is $O(t(\log t)^c)$-colorable for every $c > 0.25$, Subsequently I showed that every graph with no K_t minor is $O(t(\log\log t)^6)$-colorable. Delcourt and I improved upon this further by showing that every graph with no K_t minor is $O(t \log\log t)$-colorable. Our main technical result yields this as well as a number of other interesting corollaries. A natural weakening of Hadwiger's Conjecture is the so-called Linear Hadwiger's Conjecture that every graph with no K_t minor is $O(t)$-colorable. We prove that Linear Hadwiger's Conjecture reduces to small graphs. In 2005, Kühn and Osthus proved that Hadwiger's Conjecture for the class of $K_{s,s}$-free graphs for any fixed positive integer $s \geq 2$. Along this line, we show that Linear Hadwiger's Conjecture holds for the class of K_r-free graphs for every fixed r.