Combinatorial methods in algebraic geometry and commutative algebra
Méthodes combinatoires en géométrie algébrique et en algèbre commutative

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AYAH ALMOUSA, UNIVERSITY OF MINNESOTA - TWIN CITIES

Root polytopes, tropical types, and toric edge ideals

We explore generic tropical hyperplane arrangements where some of the apices of the tropical hyperplanes are "taken to infinity." We show that the resulting bounded complex gives rise to a cellular resolution for an ideal that is Alexander dual to the Stanley-Reisner ideal of a regular triangulation of a root polytope. Moreover, the Stanley-Reisner ideal of this triangulation can be seen as a squarefree initial ideal of a toric edge ideal of a bipartite graph; this key observation yields a new approach to studying homological aspects of toric edge ideals of bipartite graphs. This is joint work with Anton Dochtermann (Texas State) and Ben Smith (Manchester).

JULIETTE BRUCE, University of California, Berkeley

Multigraded regularity on products of projective spaces

Eisenbud and Goto described the Castelnuovo-Mumford regularity of a module on projective space in terms of three different properties of the corresponding graded module: its betti numbers, its local cohomology, and its truncations. For the multigraded generalization of regularity defined by Maclagan and Smith, these three conditions are no longer equivalent. I will characterize each of them for modules on products of projective spaces.

ELISA GORLA, University of Neuchatel

Sparse determinantal ideals: Groebner bases and Rees algebras

In this talk, I will introduce and discuss sparse determinantal ideals. Sparse determinantal ideals were first studied by Giusti and Merle. They are ideals of minors of a sparse generic matrix, i.e. a matrix whose entries are either zero or distinct variables. In his doctoral thesis, Adam Boocher studied their Groebner bases and initial ideals, proving that the maximal minors of a sparse generic matrix are a universal Groebner basis of the ideal that they generate. Together with A. Conca and E. De Negri, we computed the generic initial ideals of sparse determinantal ideals with respect to two different natural multigradings and proved that each of them is the only Borel-fixed point in the Hilbert scheme to which they belong. In a recent joint work with E. Celikbas, E. Dufresne, L. Fouli, K.-N. Lin, C. Polini, and I. Swanson, we leveraged the Groebner bases results to study the Rees algebras of determinantal ideals of 2xn sparse generic matrices and of their initial ideals. Our main tools are Groebner bases and SAGBI bases.

PATRICIA KLEIN, University of Minnesota

Bumpless pipe dreams encode Gröbner geometry of Schubert polynomials

Knutson and Miller established a connection between the anti-diagonal Gröbner degenerations of matrix Schubert varieties and the pre-existing combinatorics of pipe dreams. They used this correspondence to give a geometrically-natural explanation for the appearance of the combinatorially-defined Schubert polynomials as representatives of Schubert classes. In this talk, we will describe a similar connection between diagonal degenerations of matrix Schubert varieties and bumpless pipe dreams, newer combinatorial objects introduced by Lam, Lee, and Shimozono. This connection was conjectured by Hamaker, Pechenik, and Weigandt. This talk is based on joint work with Anna Weigandt.

JAKE LEVINSON, Simon Fraser University

Springer fibers and the Delta Conjecture at t=0

We introduce a family of varieties $Y_{n,\lambda,s}$ that we call the Δ-Springer varieties and that generalize the type A Springer fibers. We give an explicit presentation of the cohomology ring $H^*(Y_{n,\lambda,s})$ and show that it has an action of the symmetric group, generalizing the Springer action on the cohomology of a Springer fiber. In particular, the top cohomology group is an induced Specht module. The $\lambda = (1^k)$ case of this construction gives a compact geometric realization for the expression in the Delta Conjecture at $t = 0$. Finally, we generalize results of de Concini and Procesi on the scheme of diagonal nilpotent matrices by constructing an ind-variety $Y_{n,\lambda}$ whose cohomology ring is isomorphic to the coordinate ring of the scheme-theoretic intersection of an Eisenbud-Saltman rank variety and diagonal matrices.

This is joint work with Sean Griffin and Alex Woo.

CHRIS MANON, University of Kentucky

The combinatorics of toric flag bundles and the Mori dream space property

A Mori dream space is a normal, projective variety whose Cox ring is finitely generated. Projective toric varieties and the flag varieties of a semisimple group are perhaps the two most recognizable classes of spaces with this property. Accordingly, it's natural to ask when a combination of these spaces is a Mori dream space. We consider the case of toric flag bundles, namely bundles of flag varieties over a projective toric variety equipped with an action by the big torus. This is a subtle question, as even bundles of projective spaces over a toric variety can fail to be Mori dream spaces. We'll present a few theorems about Mori dream space flag bundles which link this property to the combinatorial geometry of spherical buildings and tropicalized linear spaces, and we'll describe how the answer in type A is related to a question about representation stability. This is joint work with Courtney George.

LEONARDO MIHALCEA, Virginia Tech

Positivity and log concavity conjectures in Cotangent Schubert Calculus

In Cotangent Schubert Calculus, the usual Schubert classes are replaced by characteristic classes of Schubert cells, such as the Chern-Schwartz-MacPherson classes in cohomology and motivic Chern classes in K theory. These are pull backs of certain Lagrangian cycles from the cotangent bundle of a flag manifold, and they may be calculated utilizing Demazure-Lusztig operators from the Hecke algebra. In this talk I will discuss several conjectures about positivity and log-concavity of cotangent Schubert classes, involving the transition matrix to the ordinary Schubert classes, and also about the structure constants from the multiplication of the cotangent Schubert classes. This reports on published and ongoing work with P. Aluffi, J. Schurmann, and C. Su.

FATEMEH MOHAMMADI, Ghent University

Toric degenerations of Grassmannians and their associated polytopes

Toric varieties are popular objects in algebraic geometry, as they can be modelled on polytopes and polyhedral fans. This is mainly because there is a dictionary between their geometric properties and the combinatorial invariants of their polytopes. This dictionary can be extended from toric varieties to arbitrary varieties through toric degenerations. In this talk, I will introduce the notion of toric degenerations which generalizes the fruitful correspondence between toric varieties and polytopes, to arbitrary varieties. There are prototypic examples of toric degenerations (of Grassmannians) which are related to Young tableaux and Gelfand-Cetlin polytopes. I will describe how to obtain such degenerations using the theory of Gröbner fans and tropical geometry. In this talk, I focus on particular combinatorial types of cones in tropical Grassmannians indexed by matching fields, whose corresponding degenerations are toric. Moreover, I will show how their associated Newton-Okounkov bodies (polytopes) are connected by combinatorial mutations. I will present several combinatorial conjectures and computational challenges around this problem.

ANDREW NEWMAN, Carnegie Mellon University

Random subcomplexes and Betti numbers of random edge ideals

The coedge ideal of an Erdős–Rényi random graph is a model for random squarefree monomial ideals. Using Hochster's formula one can study and interpret properties of the resulting random monomial ideal in terms of the topology of the flag complex of the random graph. By applying methods from stochastic topology we prove sharp bounds on the regularity and projective dimension of random coedge ideals in a probability regime where the Krull dimension is bounded. This is joint work with Anton Dochtermann.

SERGIO DA SILVA, McMaster University
Two approaches to geometric vertex decomposition

Geometric vertex decomposition was first introduced by Knutson-Miller-Yong to study diagonal degenerations of Schubert varieties. Later results on the topic were mostly formulated in the context of Schubert geometry, until very recent work of Klein-Rajchgot established a connection between liaison theory and geometric vertex decomposition. For homogeneous Cohen-Macaulay ideals, being geometric vertex decomposable is in some sense equivalent to being glicci (i.e. is in the Gorenstein liaison class of a complete intersection). The interplay between these two theories can be used to analyze degenerations and to construct Gröbner bases. I will highlight the first approach and its application to toric ideals of graphs, and provide an overview of the second approach in the context of Hessenberg varieties.

KELLER VANDEBOGERT, University of Notre Dame
On Constructions Related to the Generalized Taylor Complex

In this talk, we extend constructions and results for the Taylor complex to the generalized Taylor complex constructed by Herzog. We construct an explicit DG-algebra structure on the generalized Taylor complex and extend a result of Katthän on quotients of the Taylor complex by DG-ideals. We introduce a generalization of the Scarf complex for families of monomial ideals, and show that this complex is always a direct summand of the minimal free resolution of the sum of these ideals. We also give an example of an ideal where the generalized Scarf complex strictly contains the standard Scarf complex.