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Local minimum spanning tree optimization

Consider the following procedure on the complete weighted graph. Start with an initial spanning subgraph H_0 and, at each step, replace a connected subgraph of H_i with its corresponding minimum-weight spanning tree to obtain H_{i+1}. By repeating this procedure, the weight of the graphs $H_0, H_1, \ldots$ decreases and, under the assumption that we do not always choose the same subgraphs, it eventually reaches the global minimum-weight spanning tree on the complete graph.

Given an instance of this procedure, say that its weight is the maximal weight of any connected subgraph we replaced at any step. In a sense, the weight of a procedure describes how "local" the changes are, where locality is measured with respect to the current graph at each step. We pose the question: what is the smallest achievable weight, optimized over all possible replacement sequences which terminate at the minimum-weight spanning tree?

We show that, for iid $\mathrm{Uniform}([0,1])$ edge weights on the complete graph K_n, this optimal weight converges to 1 as $n \to \infty$, no matter what the initial graph H_0 is. Our proof reduces the general problem to three important special cases: when the initial graph H_0 is a complete graph, a star graph, or a Hamiltonian path.

Based on joint work with Louigi Addario-Berry and Jordan Barrett.