Applied and Pure Analysis
Analyse appliquée et pure
(Org: **Chunhua Ou** and/et **Jie Xiao** (Memorial))

DAVE AMUNDSEN, Carleton University
Analysis of Transitions Across Distinct Resonant Regimes

Resonant effects are a fundamental driver underlying a broad range of natural phenomena, from fluid and acoustic to optical and biological systems. They are characterized by the amplification of an input signal at or near a fundamental resonant frequency as determined by underlying properties of the system. While the immediate implications in specific cases are well understood, the more general relationship between input and resonant response poses a much more significant challenge and depends intricately on detailed features of the underlying model systems. In particular, based on the ways in which the modal spectrum and nonlinear effects interact, it is well known that qualitatively distinct resonant regimes can emerge with vastly differing characteristics including amplitude and profile of response. Using the parametric variation of a simple one dimensional nonlinear wave model on a bounded domain, I will firstly illustrate how qualitatively distinct outcomes may arise and their contrasting features. Then, in the context of resonantly forced acoustic waves in closed, cylindrical geometries, I will show how a continuous transition between two qualitatively distinct limiting regimes can be achieved and studied in detail.

LIA BRONSARD, McMaster University
Patterns in tri-block copolymers: droplets, double-bubbles and core-shells

We study the Nakazawa-Ohta ternary inhibitory system, which describes domain morphologies in a triblock copolymer as a nonlocal isoperimetric problem for three interacting phase domains. The free energy consists of two parts: the local interface energy measures the total perimeter of the phase boundaries, while a longer-range Coulomb interaction energy reflects the connectivity of the polymer chains and promotes splitting into micro-domains. We consider global minimizers on the two-dimensional torus, in a limit in which two of the species have vanishingly small mass but the interaction strength is correspondingly large. In this limit there is splitting of the masses, and each vanishing component rescales to a minimizer of an isoperimetric problem for clusters in 2D. Depending on the relative strengths of the coefficients of the interaction terms we may see different structures for the global minimizers, ranging from a lattice of isolated simple droplets of each minority species to double-bubbles or core-shells. This represents work with S. Alama, X. Lu, and C. Wang.

GOONG CHEN, Math. Dept., Texas A&M University
Modeling and Computation of Modal Analysis of Coronavirus

In this talk, we present our preliminary study on the modeling and supercomputer simulation of the normal modes of vibration of a coronavirus. The virus is modeled as an elastodynamic continuum. We take "samples" of coronavirus from the Internet resources. The vibratory mode shapes, as shown from post-processed supercomputer results as videos, manifest the fundamental motions from a small number of spikes to those of a higher number of spikes. As the mode sequential order increases, one can see more "breathing modes" in occurrence. At present, we are attempting to incorporate the effects of fluids (such as blood or body fluids) in the model and investigate how such vibratory motions lead to coupled motions between two or more viruses. All the modal analysis of virus vibratory motions will be visualized by video animations. Their significance is also being investigated.

WENGU CHEN, Institute of Applied Physics and Computational Mathematics
Reconstruction of Signals and Images by Prior Information

Abstract: In this talk, we consider the reconstruction of signals and images by prior information from incomplete or degraded observations. The priors includes sparsity, low-rankness, signal support information.

Key Words: Compressed sensing, RIP, Sparse representation, Weighted norm minimization.

MOHAMMAD EL SMAILY, University of Northern British Columbia
Optimal initial data for an RD model with drift

We consider a reaction-diffusion model with a drift term in a bounded domain. Given a time T, we prove the existence and uniqueness of an initial datum that maximizes the total mass $\int_\Omega u(T,x)\mathrm{d}x$ in the presence of an advection term. In a population dynamics context, this optimal initial datum can be understood as the best distribution of the initial population that leads to a maximal the total population at a prefixed time T. We also compare the total masses at a time T in two cases: depending on whether an advection term is present in the medium or not. We prove that the presence of a large enough advection enhances the total mass. This talk is based on joint work with Omar Abdul Halim from UNBC.

YUNHUI HE, The University of British Columbia
A closed-form multigrid smoothing factor for an additive Vanka-type smoother applied to the Poisson equation

We consider an additive Vanka-type smoother for the Poisson equation discretized by the standard finite difference centered scheme. Using local Fourier analysis, we derive analytical formulas for the optimal smoothing factors for two types of smoothers, called vertex-wise and element-wise Vanka smoothers, and present the corresponding stencils. Interestingly, in one dimension the element-wise Vanka smoother is equivalent to the scaled mass operator obtained from the linear finite element method, and in two dimensions the element-wise Vanka smoother is equivalent to the scaled mass operator discretized by bilinear finite element method plus a scaled identity operator. Based on these discoveries, the mass matrix obtained from finite element method can be used as an approximation to the inverse of the Laplacian, and the resulting mass-based relaxation scheme features small smoothing factors in one, two, and three dimensions. Advantages of the mass operator are that the operator is sparse and well conditioned, and the computational cost of the relaxation scheme is only one matrix-vector product; there is no need to compute the inverse of a matrix. These findings may help better understand the efficiency of additive Vanka smoothers and develop fast solvers for numerical solutions of partial differential equations.

This work is joint with Chen Greif.

RENJIN JIANG, Center for applied Math., Tianjin University
Some recent progress on Riesz transform on manifolds

In this report, we shall report some recent progress regarding progress on Riesz transform on manifolds, especially where the lower Gaussian bound for the heat kernel fails. We shall divide the situation into three cases: 1) $1 < p < 2$, 2) $2 < p < dim$, 3), $dim \le p < \infty$. In these cases the Riesz transform has substantially different behaviors.

THEODORE KOLOKOLNIKOV, Dalhousie
Modelling of disease spread through heterogeneous population

We present a simple model of disease spread that incorporates spatial variability in population density. Starting from first principles, we derive a novel PDE with state-dependent diffusion. Consistent with observations, this model exhibits higher infection rates in the areas of higher population density. The model also exhibits an infection wave whose speed varies with population density. In addition, we demonstrate the possibility of super-diffusive propagation of infection, whereby an infection can "jump" across areas of low population density towards the areas of high population density. Finally, a case study of coronavirus spread in the Canadian province of Nova Scotia is presented with qualitatively similar features as our model, including density-dependent infection rates and infection that jumps across main population centers.

NGUYEN LAM, Memorial University of Newfoundland-Grenfell Campus
Best constants, optimizers and the stability of Uncertainty Principles

In this talk, we discuss the sharp constants and optimizers of the uncertainty principles and the Caffarelli-Kohn-Nirenberg inequalities for scalar functions as well as for curl-free vector fields. We also address quantitative stability results for the Heisenberg-Pauli-Weyl uncertainty principle.

MING MEI, Champlain College Saint-Lambert & McGill University
Subsonic / supersonic / transonic steady-states for Euler-Poisson equations with sonic boundary

In this talk, we present a series of our recent studies on Euler-Poisson equations with sonic boundary for semiconductor models. Our research recognizes that the location of doping profile is the crucial mechanism for the system possessing physical solutions or not, and the size of relaxation time plays the important role for the existence of shock/smooth transonic steady-states. We will show the criteria for the existence/non-existence of all physical solutions, as well as their regularities.

ZHONGWEI SHEN, University of Alberta
Long transient dynamics in stochastic systems

Transient dynamics, often observed in multi-scale systems, are roughly defined to be the interesting dynamical behaviours that display over finite time periods. For a class of randomly perturbed dynamical systems that arise in chemical reactions and population dynamics, and that exhibit persistence dynamics over finite time periods and extinction dynamics in the long run, we use quasi-stationary distributions (QSDs) to rigorously capture the transient states governing the long transient dynamics. We study the noise-vanishing concentration of the QSDs to gain information about the transient states and investigate the dynamics near transient states to understand the transient dynamical behaviours as well as the global multiscale dynamics.

ERIK TALVILA, University of the Fraser Valley
The continuous primitive integral

The space of all Schwartz distributions, $\mathcal{D}'(\mathbb{R})$, is too large for a viable theory of integration. However, by looking for appropriate Banach spaces of distributions we can define integration processes that have many useful properties.

Primitives for the Lebesgue integral are absolutely continuous. If we take primitives to be merely continuous we obtain an integral that includes the Lebesgue and Henstock–Kurzweil integrals. Define the set of primitives $\mathcal{B}_c(\mathbb{R}) = \{F : \overline{\mathbb{R}} \to \mathbb{R} \mid F(-\infty) = 0, F \text{ is continuous on } \overline{\mathbb{R}}\}$, where $\overline{\mathbb{R}} = [-\infty, \infty]$ is the extended real line. Then $\mathcal{B}_c(\mathbb{R})$ is a Banach space under the uniform norm. Define the space of integrable distributions by taking the distributional derivative of the primitives: $\mathcal{A}_c(\mathbb{R}) = \{f \in \mathcal{D}'(\mathbb{R}) \mid f = F' \text{ for some } F \in \mathcal{B}_c(\mathbb{R})\}$. The definition of the integral is based on the fundamental theorem of calculus: If $f \in \mathcal{A}_c(\mathbb{R})$ then $\int_a^b f = F(b) - F(a)$ where $f = F'$ for a unique primitive $F \in \mathcal{B}_c(\mathbb{R})$. The Alexiewicz norm of f is $\|f\| = \|F\|_\infty$ and this makes $\mathcal{A}_c(\mathbb{R})$ into a Banach space that is isometrically isomorphic to $\mathcal{B}_c(\mathbb{R})$. This space is the completion of $L^1(\mathbb{R})$ in the Alexiewicz norm and is the smallest Banach space that contains all functions that have conditionally convergent integrals. Features useful in applications, such as a Hölder inequality, convergence theorems, convolution, and integration by parts will be discussed.

LIN WANG, University of New Brunswick
Dynamics of Intraguild Predation Models

In this talk, I will present two intraguild predation models and discuss their dynamics. One is an ODE model and the other is a DDE model. Detailed stability and bifurcation analysis show that IGP models exhibit very much dynamics such as multistability, chaos and stability switches.

MICHAEL WARD, UBC
Localized Spot Patterns for Reaction-Diffusion Systems in 3-D

Localized spot patterns, where one or more solution components concentrates at certain points in the domain, are a common class of localized pattern for reaction-diffusion systems, and they arise in a wide range of modeling scenarios. Although there is a rather well-developed theoretical understanding for this class of localized pattern in 1-D and 2-D, a theoretical study of such patterns in a 3-D setting is, largely, a new frontier. We present some new results for the existence, linear stability, and dynamics of such localized patterns for the 3-D Gierer Meinhardt model. Depending on the parameter range, spot patterns can undergo competition instabilities, leading to spot-annihilation events, or shape-deforming instabilities triggering spot self-replication events. In the absence of these instabilities, the spot locations evolve slowly towards their equilibrium locations according to an ODE gradient flow, which is determined by a discrete energy involving the reduced-wave Green's function. The central role of a certain core problem, which characterizes the profile of a localized spot, on the solution behavior is emphasized. Open problems for localization on higher co-dimension structures, such as stripes and filaments, are discussed.

JUNCHENG WEI, University of British Columbia
Stability of Sobolev Inequality

We consider the stability of Sobolev inequality at the critical point level. Suppose $u \in \dot{H}^1(\mathbb{R}^n)$. In a seminal work, Struwe proved that if $u \geq 0$ and $\Gamma(u) := \|\Delta u + u^{\frac{n+2}{n-2}}\|_{H^{-1}} \to 0$ then $dist(u, \mathcal{T}) \to 0$, where $dist(u, \mathcal{T})$ denotes the $\dot{H}^1(\mathbb{R}^n)$-distance of u from the manifold of sums of Talenti bubbles. Ciraolo, Figalli and Maggi obtained the first quantitative version of Struwe's decomposition with one bubble in all dimensions, namely $dist(u, \mathcal{T}) \leq C\Gamma(u)$. For Struwe's decomposition with two or more bubbles, Figalli and Glaudo showed a striking dimensional dependent quantitative estimate, namely $dist(u, \mathcal{T}) \leq C\Gamma(u)$ when $3 \leq n \leq 5$ while this is false for $n \geq 6$. In this talk, I will present our estimates in higher dimensions:

$$dist(u, \mathcal{T}) \leq C \begin{cases} \Gamma(u)\,|\log \Gamma(u)|^{\frac{1}{2}} & \textit{if}\, n = 6, \\ |\Gamma(u)|^{\frac{n+2}{2(n-2)}} & \textit{if}\, n \geq 7. \end{cases}$$

Furthermore, we show that this inequality is sharp. Extensions to Caffarelli-Kohn-Nirenberg inequalities, harmonic map inequality and half-harmonic map inequality will also be discussed.

DEPING YE, Memorial University
The L_p Minkowski problem for log-concave functions

The study of the geometric theory for log-concave functions has received extensive attention recently. Such a theory can be viewed as the analytic lifting of the geometric theory of convex bodies (convex compact sets with nonempty interiors in $\mathbb{R}^n$).

In this talk, we will discuss how an L_p theory of log-concave functions can be developed which builds up a framework of the L_p theory of log-concave functions. In particular, we will explain the L_p Asplund sum of log-concave functions, discuss a variational formula which can be used to derive the L_p surface area measures for log-concave functions, talk about the related L_p Minkowski problems, and present our solutions to this problem.

WEN YUAN, Beijing Normal University
Brezis–Van Schaftingen–Yung Formulae in Ball Banach Function Spaces

Let X be a ball Banach function space on $\mathbb{R}^n$. In this talk, under the mild assumption that the Hardy–Littlewood maximal operator is bounded on the associated space X' of X, we show that, for any $f \in C_c^2(\mathbb{R}^n)$,

$$\sup_{\lambda\in(0,\infty)} \lambda \left\| \left| \left\{ y \in \mathbb{R}^n :\ |f(\cdot) - f(y)| > \lambda |\cdot - y|^{\frac{n}{q}+1} \right\} \right|^{\frac{1}{q}} \right\|_X \sim \|\nabla f\|_X$$

with the positive equivalence constants independent of f, where $q \in (0, \infty)$ is an index depending on the space X. Particularly, when $X := L^p(\mathbb{R}^n)$ with $p \in [1, \infty)$, the above estimate holds for any given $q \in [1, p]$, which when $q = p$ is exactly a recent surprising formula of H. Brezis, J. Van Schaftingen, and P.-L. Yung. It also enables us to establish new fractional Sobolev and

Gagliardo–Nirenberg inequalities in various function spaces, including Morrey spaces, mixed-norm Lebesgue spaces, variable Lebesgue spaces, weighted Lebesgue spaces, Orlicz spaces.

KEXUE ZHANG, Queen's University
Event-Triggered Impulsive Control for Nonlinear Systems

Impulsive control is a control paradigm that uses impulses that are state abrupt changes over negligible time periods to control dynamic systems. Most of the existing results on impulsive control problems focus on time-triggered control strategies. More specifically, the moments when the impulses happen, normally called impulse times, are pre-scheduled which makes time-triggered control strategies simple to implement. To improve the impulsive control efficiency, event-triggered impulsive control has been successfully developed recently, the idea of which is to determine the impulse times or the instants of updating the control signals by a certain event that occurs only when the system dynamics violates a well-designed triggering condition. This talk focuses on the impulsive stabilization of nonlinear systems. We propose two types of event-triggering algorithms to update the impulsive control signals with actuation delays. The first algorithm is based on continuous event detection, while the second type makes decisions about updating the impulsive control inputs according to periodic event detection. Sufficient conditions are derived to ensure asymptotic stability of the impulsive control systems with the designed event-triggering algorithms. Lower bounds of the time period between two consecutive events are also obtained so that the closed-loop impulsive systems are free of Zeno behavior. This is joint work with Elena Braverman (University of Calgary).