Asymptotic Analysis, Orthogonal Polynomials, and Special Functions
Analyse asymptotique, polynômes orthogonaux et fonctions spéciales

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GEORGE E. ANDREWS, Pennsylvania State University
Chebyshev polynomials and compositions

The theory of compositions of integers has mostly been relegated to the very basic aspects of combinatorics. The object of this paper is to reveal their close relation to the Chebyshev polynomial Tn(x) and Un(x). As a result, interesting combinatorial questions arise for compositions that have not been examined previously.

BRUCE C. BERNDT, University of Illinois at Urbana-Champaign
Infinite Series Identities Involving a Large Class of Arithmetical Functions and Bessel Functions

We consider arithmetical functions generated by Dirichlet series satisfying "Hecke's functional equation," in the terminology of Chandrasekharan and Narasimhan in two famous papers published in 1961 and 1962. Two general classes of infinite series identities are discussed. Particular cases involve Ramanujan's tau-function $\tau(n)$; the number of representations of n as a sum of k squares, $r_k(n)$; and the sum of the kth powers of the divisors of n, $\sigma_k(n)$. Both classes of identities involve the modified Bessel function $K_\nu(z)$. This is joint work with Atul Dixit, Rajat Gupta, and Alexandru Zaharescu.

HOWARD COHL, National Institute of Standards and Technology
Special values for continuous q-Jacobi polynomials and applications

We compute special values for Askey's continuous q-Jacobi polynomials in terms of q-Racah polynomials. Then by starting with Gasper and Rahman's Poisson kernel for these polynomials, we compute new generating functions for these polynomials, those in their subfamilies such as for Rogers polynomials, and in the q to 1 limit, for Jacobi polynomials. We also study the self-duality of the corresponding q-Racah polynomials.

ROBERTO S. COSTAS-SANTOS,
Multi-integral representations for associated Legendre and Ferrers functions

For the associated Legendre and Ferrers functions of the first and second kind, we obtain new multi-derivative and multi-integral representation formulas. The multi-integral representation formulas that we derive for these functions generalize some classical multi-integration formulas. As a result of the determination of these formulae, we compute some interesting special values and integral representations for certain particular combinations of the degree and order including the case where there is symmetry and antisymmetry for the degree and order parameters. As a consequence of our analysis, we obtain some new results for the associated Legendre function of the second kind including parameter values for which this function is identically zero.

This is a Joint work with Howard S. Cohl (NIST)

DIEGO DOMINICI, Johannes Kepler Universität
Mehler-Heine type asymptotic formulas for discrete semiclassical orthogonal polynomials

In this talk, we will present a uniform approach for finding asymptotic approximations of semiclassical orthogonal polynomials in the region x=O(1). We will put special emphasis on the Generalized Charlier and Meixner families.

T. MARK DUNSTER, San Diego State University
Computation of parabolic cylinder functions with complex argument

Parabolic cylinder functions have many mathematical and physical applications. Here we study the function $U(a, z)$, which is characterised as being recessive (exponentially small) at $z = +\infty$. Algorithms for its evaluation for real z are well-established. The case z complex has important applications in asymptotic solutions of differential equations having two turning points in the complex plane, which is a planned further study.

We consider the computation of $U(a, z)$ for unrestricted (unbounded) complex z, with the parameter $a \in [0, \infty)$. Methods for its fast computation are constructed at close to double precision accuracy. To do so, uniform asymptotic expansions for large a with z unrestricted are used, and for the other (small to moderate) values of a we employ contour integral representations, small and large z expansions, along with connection formulas.

This is joint work with Ampero Gil and Javier Segura, Universidad de Cantabria, Santander, Spain.

MOURAD ISMAIL, University of Central Florida
New Orthogonal Polynomials of Askey–Wilson Type

We study two families of orthogonal polynomials. The first is a finite family related to the Askey–Wilson polynomials but the orthogonality is on the real line. A limiting case of this family is an infinite system of orthogonal polynomials whose moment problem is indeterminate. We provide several orthogonality measures for the infinite family and derive their Plancherel-Rotach asymptotics. The polynomials also satisfy second order divided difference equations.

ARNO KUIJLAARS, Katholieke Universiteit Leuven
Asymptotics of matrix valued orthogonal polynomials

I will discuss the asymptotics of matrix valued orthogonal polynomials by means of the steepest descent analysis of the associated Riemann-Hilbert problem. This is joint work with Alfredo Deaño (Madrid, Spain) and Pablo Román (Cordoba, Argentina).

ABEY LÓPEZ-GARCÍA, University of Central Florida
Spectral properties of random banded Hessenberg matrices

In this talk I will consider a class of random, banded lower Hessenberg matrices and discuss its spectral asymptotic properties. Each diagonal of the matrices is formed by i.i.d. random variables, with distributions that may be different for different diagonals. We prove convergence in expectation of the moments of the empirical spectral distribution of the matrices considered as their size tends to infinity. An important tool we use is the Hermite-Padé property for a system of resolvent functions of the limiting Hessenberg operator. This is a joint work with Vasiliy A. Prokhorov.

GERGŐ NEMES, Tokyo Metropolitan University and Alfréd Rényi Institute of Mathematics
A proof of a conjecture of Elbert and Laforgia on the zeros of cylinder functions

We prove the enveloping property of the known divergent asymptotic expansion of the large real zeros of the cylinder functions, and thereby answering in the affirmative a conjecture posed by Elbert and Laforgia in 2001 (*J. Comput. Appl. Math.* **133** (2001), no. 1–2, p. 683). The essence of the proof is the construction of an analytic function that returns the zeros when evaluated along certain discrete sets of real numbers. By manipulating contour integrals of this function, we derive the asymptotic expansion of the large zeros truncated after a finite number of terms plus a remainder that can be estimated efficiently. The conjecture is then deduced as a corollary of this estimate.

WALTER VAN ASSCHE, KU Leuven, Belgium
Multiple orthogonal polynomials and the number π

It is well known that π is an irrational number (and transcendental), but it is still not known how well it can be approximated by rational numbers. Last year Zeilberger and Zudilin (2020) found the best upper bound so far: the measure of irrationality of π is bounded from above by 7.103205334137. They improved an earlier upper bound of Salikhov from 2008, and before him the best upper bound was obtained by Hata (1993). These upper bounds were obtained by analyzing certain integrals of rational functions over contours in the complex plane. In my talk I will show that these integrals are closely related to an Hermite-Padé approximation problem for a pair of Markov functions. We will investigate this Hermite-Padé approximation in some detail, in particular the corresponding multiple orthogonal polynomials, and we do the required asymptotic analysis using the steepest descent method.

LUC VINET, CRM, Université de Montréal
Sklyanin algebras and orthogonal polynomials

It will be shown how algebras of Sklyanin type can be obtained from special Heun operators on various grids. Some examples will be treated. The orthogonal polynomials providing bases for representations of these algebras will be identified. Finite-dimensional representations obtained from truncations will be seen to yield interpretations of so-called para-polynomials.

XIANG-SHENG WANG, University of Louisiana at Lafayette
Error bounds for the asymptotic expansions of the Hermite polynomials

We present explicit and computable error bounds for the asymptotic expansions of the Hermite polynomials with the Plancherel–Rotach scale. Three cases, depending on whether the scaled variable lies in the outer or oscillatory interval, or it is the turning point, are considered separately. We introduce the "branch cut" technique to express the error terms as integrals on the contour taken as the one-sided limit of curves approaching the branch cut. This new technique enables us to derive simple error bounds in terms of elementary functions. We also provide recursive procedures for the computation of the coefficients appearing in the asymptotic expansions.